LENGTH OF CLOSED FISHTAIL-SHAPED TURNS AND HEADLAND WIDTH WHEN PLOUGHING WITH A REVERSIBLE PLOUGH IN AN IRREGULARLY SHAPED FIELD
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ДЪЛЖИНА НА ЗАТВОРЕНИ ГЪБОВИДНИ ЗАВОИ И ШИРОЧИНА НА ИВИЦАТА ЗА ЗАВИВАНЕ ПРИ ОРАН С ОБРЪЩАТЕЛЕН ПЛУГ В ПОЛЕ С НЕПРАВИЛНА ФОРМА

Krasimir TRENDAFILOV
Trakia University – Stara Zagora, Faculty of Technics and Technologies – Yambol, Department of Mechanical Engineering/ Bulgaria
Tel: +359 887 708 603; E-mail krasimir.trendafilov@trakia-uni.bg
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ABSTRACT
The efficiency of agricultural operations depends on the length of non-working passes in the headlands. Therefore, the aim is to minimize both turn length and headland width. Various methods are used to construct turn trajectories, allowing them to be compared under specific conditions and enabling the most efficient movement pattern to be selected. This article examines closed fishtail-shaped turns composed of arcs with a given radius and straight-line segments. Analytical relationships were derived to determine the headland width and the length of closed fishtail-shaped turns for different travel directions of a machine-tractor unit equipped with a mounted reversible plough in an irregularly shaped field. It was found that, at a positive angle between the travel direction of the unit and the field boundary, movement of the unit across the field from right to left results in shorter closed fishtail-shaped turns and requires a narrower headland than movement from left to right.

РЕЗЮМЕ
Ефективността на извършване на земеделските операции зависи от дължината на неработните ходове в ивиците за завиване. Стремежът е дължината на завоите и широчината на ивицата за завиване да бъдат минимални. За построяване на траекторията на завоите се използват различни методи, които позволяват те да бъдат сравнявани при определени условия и да бъде избиран най-ефективният начин на движение. В настоящата статия са разгледани затворени гъбовидни завои изградени от дъги с определен радиус и прави линии. Изведени са аналитични зависимости за определяне на широчината на ивицата за завиване и дължината на гъбовидни затворени завои при различни направления на движение на машинно-тракторен агрегат с навесен обръщателен плуг в поле с неправилна форма. Установено е, че при положителен ъгъл между посоката на движение на агрегата и границата на полето и движение на агрегата по полето отдясно наляво гъбовидните затворени завои са с по-малка дължина и изискват по-тясна ивица за завиване в сравнение с движението отляво надясно.

INTRODUCTION
To increase the efficiency of the movement of autonomous machine-tractor units in the field, it is necessary to determine the most suitable trajectories of movement not only inside the field, but also in the headlands. If when moving in the field, efficiency is determined mainly by the direction and sequence of work movements, then when moving in the headlands, a choice must be made between different types of turns. The goal is to reduce the time for making turns, their length and the headland width. This, in turn, will lead to an increase in the productivity of the units and a decrease in soil compaction in the headlands. Ward et al. divided the headland into three zones – transition area, turning area and area located near to the field edge. The authors found that yields in these zones were lower than yields in the field. The main characteristic of the turning area was traffic, and this zone consistently reported worse yields than those in the field area. The outer zone also yielded significantly less, indicating that there were factors other than traffic that had an impact (*Ward et al., 2020*).

To select the most effective way of turning, the authors compare different types of turns, using the following approaches to build the turns:

1. The turns consist of primitives – straight lines and arcs with a certain radius. This allows the turns to be compared without considering factors such as driving speed and wheel turning speed.

Jeon et al. consider a stretched turn and a closed fishtail turn, which are created from arcs with the minimum turning radius of the tractor and straight lines. When performing a closed fishtail turn, the shape of the turn does not change depending on the angle between the direction of the working move and the field boundary, which leads to an additional path when entering or leaving the headland, depending on this angle (*Jeon et al., 2021*). In another study, the turns were built in the same way and again the shape of the turns was not adjusted to the angle between the working move and the field boundary. This results in a larger headland width *(Wang et al., 2025*). The authors point out that the length of the non-working move decreases by 18%, which is due to the different way of moving in the field.

Han et al. conducted a simulation study of three types of turns in the headland. In the proposed algorithm, the turns are built from curves with a certain turning radius of the tractor and straight sections. The highest field efficiency is achieved when performing a fishtail turn (X-shaped) due to the shorter path in the headland, which leads to less total field processing time. The authors recommend this turn for fields with a larger width, while for small widths (30 m) the efficiency is higher when performing a stretched turn (C-shaped) (*Han et al., 2019*).

An algorithm for performing a closed fishtail turn with an arcuate reverse is presented in (*Wang and Noguchi, 2018*). Unlike other algorithms, the turning radii in the two directions are different. The authors indicate that compared to a fishtail turn with a straight reverse movement, their proposed movement algorithm provides a smaller lateral deviation (3.9 cm), and the time and trajectory of the turn are reduced by 17% and 21%, respectively. The headland width is also smaller.

Chen et al. derived relationships for the turn length and turning time in the headland for four types of turns, but the type of turn depends on the angle between the travel direction and the field boundary, as well as on the direction in which the turn is performed. According to the authors, the largest reduction in turn length, the turning time and the headland width is achieved for the semicircular turn to the left (*Chen et al., 2023*).

2. The turns are made up of straight lines, arcs with a certain radius and connecting curves with a variable radius

When the turns are made up of arcs with a minimum radius and straight lines, it is necessary for the unit to stop or the turning of the wheels to be done very quickly, which leads to a large jerk of the machine. Therefore, Boryga proposes to use trigonometric transition curves to perform turns in the headland. They have a gradually changing curvature, which provides a smooth transition between the segments of the trajectory. This reduces sudden changes in direction and improves the dynamics of movement (*Boryga et al., 2025*).

Sabelhaus et al. analysed three forward turns in which clothoids were used to connect straight-line segments and circular arcs *(Sabelhaus et al., 2015)*. A clothoid has a curvature that varies progressively up to the reciprocal of the minimum turning radius, thereby providing a smooth transition between the other elements of the trajectory. Sabelhaus et al. also examined seven different headland turns constructed using clothoids. The authors indicated that the headland width is smallest in the fishtail turn, and the turn length and the time of its execution depend on the distance between the working moves. At a short distance of 3 m, the fishtail turn has the shortest turn length and the shortest driving time. While at 10 m the values of these indicators are lower in the Ω-turn (*Sabelhaus et al., 2013*).

Koc presents an analytical approach to modeling the curvature of the connecting curve using differential equations. The method can be applied to identify various types of transition curves. The author gives examples of solving the problem involving linear change of curvature and the curvature in polynomial and trigonometric form (*Koc, 2014*).

3. Turns involve curves with variable radius and follow a guiding curve that usually copies the field boundary

An algorithm is used to determine the field boundary based on images, then a reference line for turning is generated at a certain distance from the field boundary, and the turning trajectory is designed based on Bezier curves. The authors found that when turning at a lower speed, there is less lateral deviation and better trajectory tracking effect, but the turning time is longer (*Gao et al., 2023*).

Another study presented a field boundary detection algorithm based on DeepLabv3+, which also uses Bezier curves to calculate the turning trajectory. It was found that the robot can successfully perform a turn with a maximum deviation of 0.09 m, demonstrating precise navigation control in complex agricultural environments (*Wu et al., 2025*).

A similar algorithm is described in the patent (*Kakkar et al., 2021*). The turn is made along a guide line, which has the shape of the field boundary. A different turn trajectory can be selected, which can be displayed on the screen. The algorithm can select a trajectory according to various optimization criteria - road length, fuel economy, etc.

4. The turns are composed of two symmetrical curves described by polynomials

Boryga et al. propose an algorithm for planning a trajectory in which the movement proceeds along two transitional curves, constituting the so-called transition bi-curve. The authors consider turns of 90, 180, 270 and 360 degrees. According to them, the trajectories, the curvature of which is described by a polynomial of the third degree are the most favourable. They are characterized by the smallest absolute values of maximal acceleration and jerk and to lack jerk discontinuity (*Boryga, et al., 2020*).

Since the low efficiency of turning (turn length, time for its execution, headland width) is caused by the large turning radius of the tractor, a headland turning system has been created, including a steering auxiliary wheel in front of the tractor. This provides a large turning angle of ± 180º. This way of turning reduces the time for turning by about 50%, the turn length by about 80%, and the headland width by an average of about 50% for different types of turns (*Yang et al., 2022*). Such a turning system is more effective for units with a small and medium working width - up to 9 – 11 m (*Trendafilov and Delchev, 2018*). Systems with a fifth wheel on the tractor have not yet gained popularity.

The steering system built into the tractor can be supplemented with control of the breaking force of the wheels when turning. This will result in a reduction in the turning radius by about 15% to 35% compared to a manually operated vehicle without breaking (*Bharadwa, et al., 2021*).

Fishtail turns usually have the smallest length and width of headland. Therefore, they are preferred when making turns by implements with mounted machines. The aim of this study is to determine the headland width and the length of fishtail closed turns performed by a reversible plough in different directions of movement in an irregularly shaped field.

MATERIALS AND METHODS

Previous publications examined the movement of trailed and semi-mounted reversible ploughs in the headlands of an irregularly shaped field (*Trendafilov, 2010; Trendafilov, 2017*). In these cases, the turns were performed in forward motion. Trendafilov derived relationships for determining the turn length and headland width when performing open fishtail-shaped turns with reverse movement *(Trendafilov, 2025)*. The same methodology is used in the present article and is described again for the reader’s convenience.

Fig. 1 shows a diagram of the machine-tractor unit. Its designations are:

- α is the angle between the direction of movement of the unit and the headland. It varies between 0º and 90º. It is assumed that the angle is positive when it is on the right side of the unit when moving towards the headland (Fig. 1a), and negative – on the left side of the unit (Fig. 1b);

- B – the working width of the unit;

- l_a– the kinematic length of the unit;

- l_n – the distance from the center of the unit (the middle of the rear axle of the tractor) to the top of the share of the first plough body;

- g – the distance from the axis of symmetry of the tractor to the top of the share of the last plough body;

- y – the distance from the tractor's axis of symmetry to the end of the share of the first plough body (or to the inner edge of the wheel of the tractor, which moves in the furrow left during the previous working stroke);

- R – the turning radius of the unit;

- M – the distance between the outer edges of the rear wheels of the tractor;

- H – the longitudinal base (distance between the front and rear axles) of the tractor;

- E – the headland width.

A closed fishtail turn is considered with two directions of movement of the machine-tractor unit in the field – from left to right and from right to left. The turn is made up of three circular segments with a radius equal to the minimum turning radius of the unit. The turn length in the headland includes the length of the circular segments and the length of the rectilinear path travelled by the unit until reaching the inner border of the headland. The headland width is determined by summing straight segments, the length of which can be determined from the parameters of the unit. The headland width and the turn length are determined for an interval of change of angle α from 10º to 90º, since below 10º a headland is not needed.

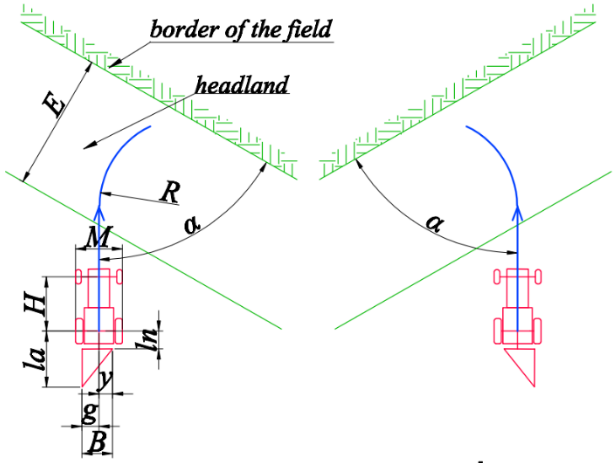

Fig.1 - Scheme of the machine-tractor unit and angle between its direction of movement and the field boundary (*Trendafilov, 2025*)

a – positive angle; b – negative angle

Only the case in which angle α is positive is considered, since a symmetric image is obtained at a negative angle. As indicated by *Trendafilov (2010)*, the relationships are the same, but their positions are interchanged. Thus, the relationships for movement across the field from left to right at a positive angle apply to movement across the field from right to left at a negative angle, while the relationships for movement from right to left at a positive angle α apply to movement from left to right at a negative angle.

Using the obtained relationships, the headland width and turn length were determined for a ploughing unit with the following parameters: $H = 2\ m$; $M = 1.8\ m$; $R = 3.2\ m$; $B = 1.2\ m$; $l_a = 2\ m$; $l_n = 0.6\ m$; $y = 0.5\ m$; $g = 0.7\ m$. The results are presented graphically and analysed.

RESULTS

Closed fishtail turn - movement from left to right

As shown in Fig. 2a, the turn is composed of three circular arcs, whose central angles satisfy the following relationship:

$$\beta_1 + \beta_2 + \beta_3 = 180° \tag{1}$$

Hence, the turn length is obtained as:

$$l_T = \pi R \tag{2}$$

The width of headland E can be determined by summing the segments of known length:

$$x = l_a . \sin\alpha + (R - g)\cos\alpha - R\cos(\alpha + \beta_1) \tag{3}$$

$$y = H . \sin(\alpha + \beta_1) \tag{4}$$

$$z = -0.5M . \cos(\alpha + \beta_1) \tag{5}$$

Therefore, the headland width is obtained as:

$$E = x + y + z = l_a \sin\alpha + (R - g)\cos\alpha + H . \sin(\alpha + \beta_1) - (R + 0{,}5M)\cos(\alpha + \beta_1) \tag{6}$$

Angle β_1 is determined from triangles O_1CO_2 and O_2DO_3. The following equations are used:

$$2R\cos\beta_1 + 2R\cos\beta_3 = 2R - B \tag{7}$$

$$2R\sin\beta_1 + l_a + l_n = 2R\sin\beta_3 \tag{8}$$

Thus angle β_1 is obtained as:

$$\beta_1 = \cos^{-1}\left[\frac{2R - B}{4R\cos\left[\tan^{-1}\left(\frac{l_a + l_n}{2R - B}\right)\right]}\right] - \tan^{-1}\left(\frac{l_a + l_n}{2R - B}\right) \tag{9}$$

When angle α decreases without changing angles β, the headland is bounded by the front left wheel of the tractor, which reaches the field boundary, and by the rear plough body. At a certain value of the angle α, the headland is additionally bounded, besides the last plough body, by the rear right wheel of the tractor at the end of its reverse movement (Fig. 2b). This condition is necessary to prevent the tractor from entering the already worked part of the field. In this case, the segments used to determine the headland width have the following lengths: y – determined using Eq. (4); z – determined using Eq. (5); and

$$e = (R + 0{,}5M)\cos(\beta_3 - \alpha) \tag{10}$$

$$d = R\cos(\alpha + \beta_1) \tag{11}$$

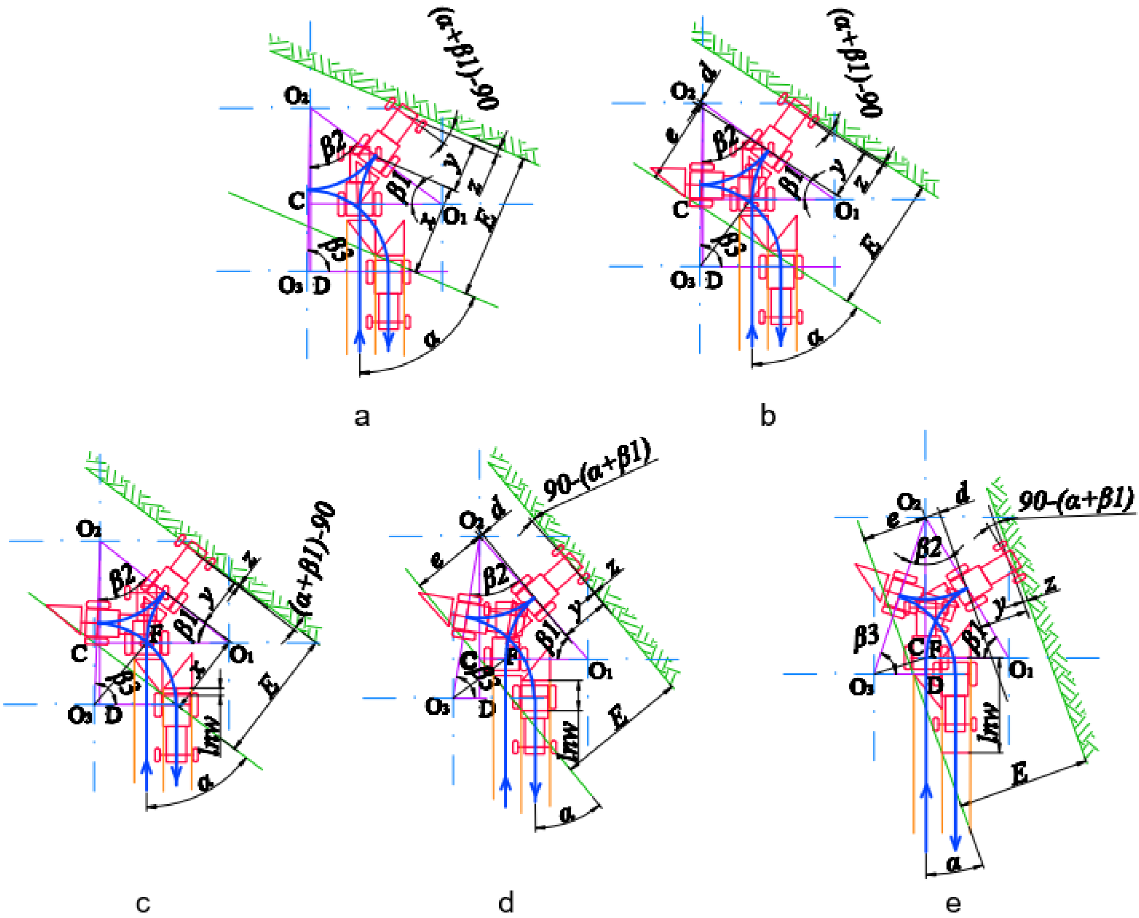

Fig. 2 - Scheme of a closed fishtail turn when moving from left to right

The headland width is given by:

$$E = e - d + y + z = (R - 0{,}5M)\cos(\alpha + \beta_1) + H.\sin(\alpha + \beta_1) + (R + 0{,}5M)\cos(\beta_3 - \alpha) \tag{12}$$

From Eqs. (7) and (8), angle β_3 is determined as follows:

$$\beta_3 = \tan^{-1}\left(\frac{l_a + l_n}{2R - B}\right) + \cos^{-1}\left[\frac{2R - B}{4R\cos\left[\tan^{-1}\left(\frac{l_a + l_n}{2R - B}\right)\right]}\right] \tag{13}$$

After equating Eqs. (6) and (12), the value of angle α at which the transition between the two equations occurs is determined as follows:

$$\alpha = \tan^{-1}\left[\frac{(R + 0{,}5M)\cos\beta_3 - (R - g) + 2R\cos\beta_1}{l_a + 2R\sin\beta_1 - (R + 0{,}5M)\sin\beta_3}\right] \tag{14}$$

Both front wheels of the tractor reach the end of the field simultaneously when:

$$\alpha + \beta_1 = 90 \tag{15}$$

For values of angle α lower than:

$$\alpha = \tan^{-1}\left(\frac{l_a + l_n}{R - B}\right) \tag{16}$$

the values of angles β change. The boundary of the headland passes through the last plough body and the rear right wheel of the tractor (Fig. 2c).

After completing the turn, the tractor must perform a straight-line non-working pass of length l_{nw} until the front plough body reaches the headland boundary. The headland width is determined using Eq. (12). The straight-line non-working pass is given by:

$$l_{nw} = l_a + l_n - (R - B)\tan\alpha \tag{17}$$

The turn length is given by:

$$l_T = \pi R + l_{nw} = \pi R + l_a + l_n - (R - B)\tan\alpha \tag{18}$$

The values of angles β are determined from triangles O_1CO_2 and O_2DO_3 using Eq. (7) and the following equation:

$$2R\sin\beta_1 + (R - B)\tan\alpha = 2R\sin\beta_3 \tag{19}$$

In this expression, angle α is used for simplification, since angle DO_3F is approximately equal to α. Thus, the angles β are obtained as follows:

$$\beta_1 = \cos^{-1}\left[\frac{2R-B}{4R\cos\left[\tan^{-1}\left(\frac{(R-B)\tan\alpha}{2R-B}\right)\right]}\right] - \tan^{-1}\left(\frac{(R-B)\tan\alpha}{2R-B}\right) \tag{20}$$

$$\beta_3 = \tan^{-1}\left(\frac{(R-B)\tan\alpha}{2R-B}\right) + \cos^{-1}\left[\frac{2R-B}{4R\cos\left[\tan^{-1}\left(\frac{(R-B)\tan\alpha}{2R-B}\right)\right]}\right] \tag{21}$$

When angle α decreases and reaches the value

$$\alpha = \tan^{-1}\left(\frac{B}{l_a - l_n}\right) \tag{22}$$

the headland boundary becomes parallel to the beam carrying the plough bodies (Fig. 2d). At a smaller angle, the headland boundary is no longer determined by the last plough body, but by the first plough body when the unit enters the headland and by the rear right wheel of the tractor (Fig. 2e). The front right wheel of the tractor reaches the field boundary. The lengths of the segments used to determine the headland width are as follows: e, determined using Eq. (10); d, determined using Eq. (11); y, determined using Eq. (4); and

$$z = 0{,}5M.\cos(\alpha + \beta_1) \tag{23}$$

For the headland width, the following expression is obtained:

$$E = e + d + y + z = (R + 0{,}5M)\cos(\alpha + \beta_1) + H.\sin(\alpha + \beta_1) + (R + 0{,}5M)\cos(\beta_3 - \alpha) \tag{24}$$

By equating Eqs. (12) and (24), it can be seen that they are equal when condition (15) is satisfied. This corresponds to the transition between the variants shown in Fig. 2c and Fig. 2e.

The length of the straight-line non-working pass in the headland after the turn increases as angle α decreases. It is determined as follows:

$$l_{nw} = \frac{B}{\tan\alpha} + l_n - (R - B)\tan\alpha \tag{25}$$

The turn length is:

$$l_T = \pi R + l_{nw} = \pi R + \frac{B}{\tan\alpha} + l_n - (R - B)\tan\alpha \tag{26}$$

By equating Eqs. (18) and (26), the angle at which the turn lengths for the two variants are equal is obtained:

$$\alpha = \tan^{-1}\left(\frac{B}{l_a}\right) \tag{27}$$

<u>Closed fishtail turns - movement from right to left</u>

At a large angle α, the front right wheel of the tractor reaches the field boundary (Fig. 3 a). During reverse movement, the tractor does not reach the field boundary. The turn length is also determined using Eqs. (1) and (2). The lengths of the segments used to determine the headland width are as follows:

$$x = l_a\sin\alpha + (g - R)\cos\alpha + R\cos(\alpha - \beta_1) \tag{28}$$

$$y = H.\sin(\alpha - \beta_1) \tag{29}$$

$$z = 0{,}5M.\cos(\alpha - \beta_1) \tag{30}$$

The headland width is:

$$E = x + y + z = l_a\sin\alpha + (g - R)\cos\alpha + H\sin(\alpha - \beta_1) + (R + 0{,}5M)\cos(\alpha - \beta_1) \tag{31}$$

Angles β_1 and β_3 are determined using equations (9) and (13) respectively. When angle α decreases and reaches a certain value, the rear wheels of the tractor also reach the field boundary during reverse movement (Fig. 3 b). The figure shows the limiting case in which the front and rear wheels of the tractor reach the field boundary. In this case, the headland width can be determined using Eq. (31) or by using the following segments:

$$m = -(R + 0{,}5M)\cos(\alpha + \beta_3) \tag{32}$$

$$n = (R - y)\cos\alpha \tag{33}$$

$$q = l_n \sin\alpha \tag{34}$$

$$E = m + n - q = -(R + 0{,}5M)\cos(\alpha + \beta_3) + (R - y)\cos\alpha - l_n \sin\alpha \tag{35}$$

After equating Eqs. (31) and (35), angle α is obtained as follows:

$$\alpha = \tan^{-1}\left[\frac{H\sin\beta_1 - (R+0{,}5M)(\cos\beta_3 + \cos\beta_1) - (B-2R)}{H\cos\beta_1 + (R+0{,}5M)(\sin\beta_1 - \sin\beta_3) + l_a + l_n}\right] \tag{36}$$

At smaller values of angle α the headland width is determined using Eq. (35). At the angle α determined using Eq. (22), the line of all plough bodies becomes parallel to the headland boundary.

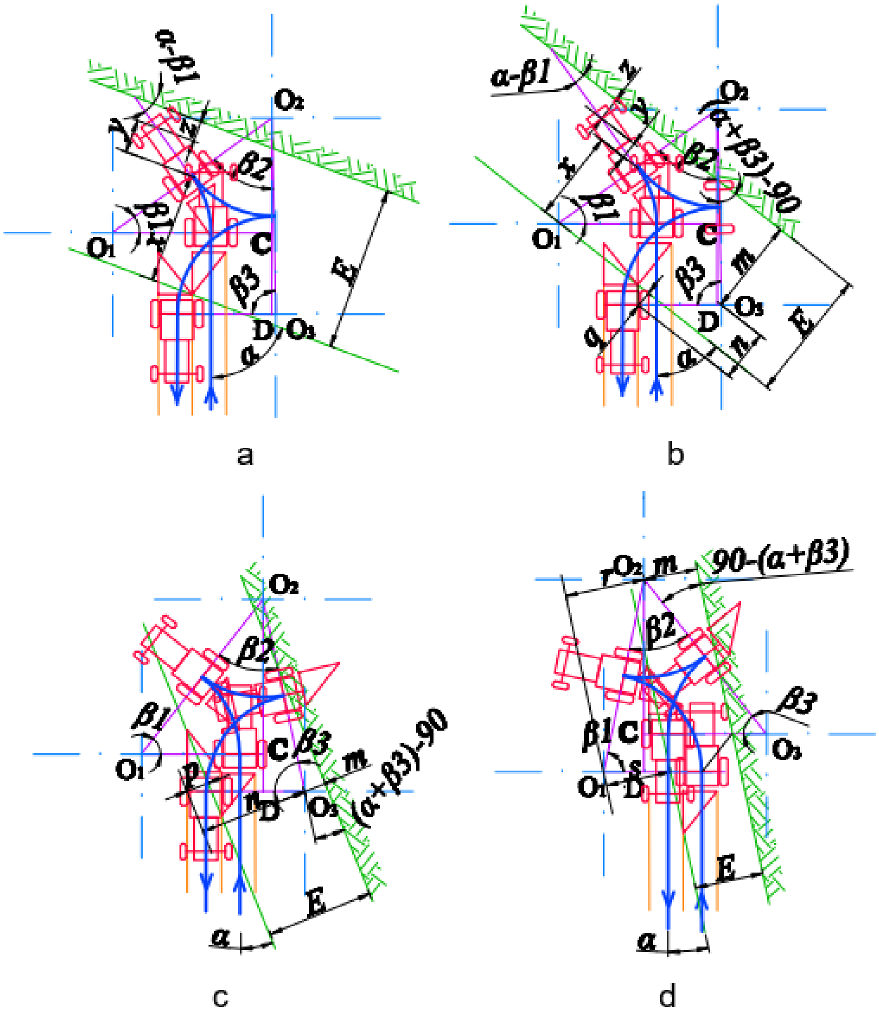

Fig. 3 - Scheme of a closed fishtail turn when moving from right to left

For values of angle α smaller than that determined by Eq. (22), the headland boundary is determined by the last plough body when the unit enters and leaves the headland (Fig. 3c). It can be seen that the front left wheel of the tractor enters the field; however, this is permissible because this part of the field has not yet been processed.

The relationships used to determine the headland width are as follows: (m), determined using Eq. (32), and:

$$p = l_a \sin\alpha \tag{37}$$

$$n = (R + g)\cos\alpha \tag{38}$$

$$E = m + n - p = -(R + 0{,}5M)\cos(\alpha + \beta_3) + (R + g)\cos\alpha - l_a \sin\alpha \tag{39}$$

Angles β vary depending on angle α and are determined using Eq. (7) and the following equation:

$$2R\sin\beta_3 - 2R\sin\beta_1 = 2l_a - \frac{B}{\tan\alpha} \tag{40}$$

The equations for determining angles β are as follows:

$$\beta_1 = \cos^{-1}\left[\frac{2R-B}{4R\cos\left[\tan^{-1}\left(\frac{2l_a-\frac{B}{\tan\alpha}}{2R-B}\right)\right]}\right] - \tan^{-1}\left(\frac{2l_a-\frac{B}{\tan\alpha}}{2R-B}\right) \tag{41}$$

$$\beta_3 = \tan^{-1}\left(\frac{2l_a-\frac{B}{\tan\alpha}}{2R-B}\right) + \cos^{-1}\left[\frac{2R-B}{4R\cos\left[\tan^{-1}\left(\frac{2l_a-\frac{B}{\tan\alpha}}{2R-B}\right)\right]}\right] \tag{42}$$

The rear axle of the tractor becomes parallel to the field boundary when:

$$\alpha + \beta_3 = 90 \tag{43}$$

As angle α decreases further, the rear right wheel reaches the field boundary (Fig. 3 d). In this case, the following relationships are used to determine the headland width:

$$m = (R + 0{,}5M)\cos(\alpha + \beta_3) \tag{44}$$

$$r = 2R\cos(\beta_1 - \alpha) \tag{45}$$

$$s = (R - g)\cos\alpha - l_a\sin\alpha \tag{46}$$

$$E = m + r - s = (R + 0{,}5M)\cos(\alpha + \beta_3) + 2R\cos(\beta_1 - \alpha) - (R - g)\cos\alpha + l_a\sin\alpha \tag{47}$$

The variants shown in Fig. 3c and Fig. 3d are equivalent at the angle:

$$\alpha = \tan^{-1}\left(\frac{(R+0{,}5M)\cos\beta_3 + R\cos\beta_1 - R}{(R+0{,}5M)\sin\beta_3 - R\sin\beta_1 - l_a}\right) \tag{48}$$

At an angle α smaller than that determined by

$$\alpha = \tan^{-1}\left(\frac{B}{2l_a}\right) \tag{49}$$

angles β are determined using Eq. (7) and the following equation:

$$2R\sin\beta_1 - 2R\sin\beta_3 = \frac{B}{\tan\alpha} - 2l_a \tag{50}$$

Their values are obtained as follows:

$$\beta_1 = \tan^{-1}\left(\frac{\frac{B}{\tan\alpha}-2l_a}{2R-B}\right) + \cos^{-1}\left[\frac{2R-B}{4R\cos\left[\tan^{-1}\left(\frac{\frac{B}{\tan\alpha}-2l_a}{2R-B}\right)\right]}\right] \tag{51}$$

$$\beta_3 = \cos^{-1}\left[\frac{2R-B}{4R\cos\left[\tan^{-1}\left(\frac{\frac{B}{\tan\alpha}-2l_a}{2R-B}\right)\right]}\right] - \tan^{-1}\left(\frac{\frac{B}{\tan\alpha}-2l_a}{2R-B}\right) \tag{52}$$

Fig. 4 shows the variation in headland width depending on the angle between the direction of the working pass and the field boundary for a specific machine-tractor unit. It can be seen that when the turn is performed from right to left, a narrower headland is required. Its width decreases as angle α decreases. When the unit moves across the field from left to right, the headland width changes only slightly and remains constant at angles below 50°.

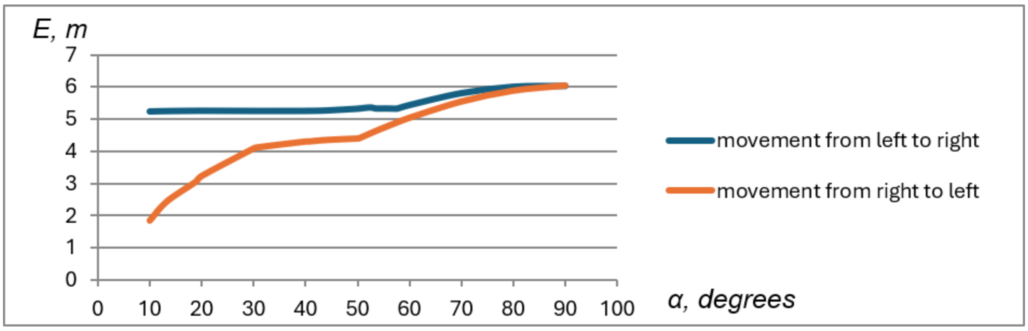

Fig. 4 - Headland width when making fishtail closed turns in different directions of movement depending on the angle α

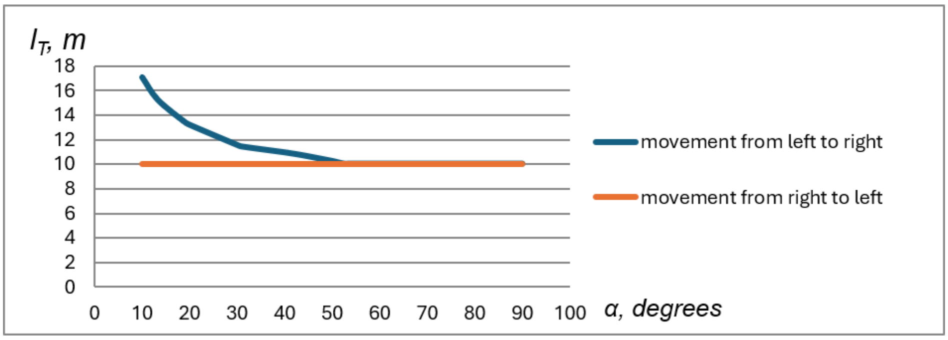

Fig. 5 - Turn length in the headland when performing fishtail closed turns in different directions of movement depending on the angle α

Fig. 5 shows the turn length for different travel directions. The turn length remains constant when the unit moves from right to left, whereas it increases significantly at smaller values of angle α when the unit moves from left to right. This is due to the presence of an additional straight-line path in the headland under this movement mode, which occurs at small values of angle α.

CONCLUSIONS

Analytical relationships were derived for determining the headland width and the length of closed fishtail-shaped turns during the movement of a machine-tractor unit equipped with a mounted reversible plough in an irregularly shaped field. It was established that, at a positive angle between the travel direction of the unit and the field boundary, movement of the unit across the field from right to left results in shorter closed fishtail-shaped turns and requires a narrower headland compared with movement from left to right. Future research should include a comparative analysis of open and closed fishtail-shaped turns in terms of turn length and headland width, and should provide relevant practical recommendations.

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