RESEARCH ON OPTIMIZING THE POWER BALANCE OF A PLOWING AGGREGATE COMPRISED OF A 180 HP TRACTOR AND A 5-MOLDBOARD PLOW
/
CERCETĂRI PRIVIND OPTIMIZAREA BILANȚULUI DE PUTERE AL UNUI AGREGAT DE ARAT FORMAT DINTR-UN TRACTOR DE 180 CP ȘI UN PLUG CU 5 TRUPIȚE

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DOI: https://doi.org/10.35633/inmateh-75-83

Keywords: *agricultural machine, tillage, traction force, mathematical modeling, power balance, optimization*

ABSTRACT
Plowing is recognized as an essential agricultural task that cannot yet easily be substituted with alternative soil processing methods due to its significance. However, it is also one of the most fuel-intensive operations. The main objective of this paper is to determine the optimal operational parameters (working speed and working width) of a plowing aggregate composed of a 180 HP tractor and a 5-moldboard plow, which ensures the full utilization of the tractor engine's power and the use of the aggregate at its maximum working capacity, respectively, achieving an optimal power balance.

REZUMAT
Lucrarea de arat este recunoscută ca fiind una încă greu de înlocuit de alte lucrări de pregătire a patului germinativ datorită importanței și utilității acesteia. În același timp, lucrarea de arat este una dintre cele mai mari consumatoare de combustibil din întreg ciclul de cultivație. Obiectivul principal al lucrării constă în determinarea parametrilor de operare optimi pentru un agregat de arat compus dintr-un tractor de 180 CP și un plug cu 5 trupițe, care să asigure utilizarea în întregime a puterii motorului tractorului și folosirea agregatului la capacitatea maximă de lucru, respectiv, realizarea un bilanț de putere optimal.

INTRODUCTION
Plowing is a crucial agricultural operation that demands significant fuel consumption. To reduce the significant amount of fuel required for seedbed preparation operations, particularly for plowing, the tractor-implement aggregates must be properly configured, and the operating parameters must be selected to ensure the full utilization of the tractor engine's power *(Șandru et al., 1983; Croitoru et al., 2017)*. Additionally, the soil processing equipment should operate at its maximum working capacity while maintaining an optimal power balance *(Hunt, 1986)*.

Studies on soil-machine interaction aim to provide scientific insights into the dynamics between tillage tools, traction devices, and the terrain they operate on. Key variables examined include the forces needed to operate tillage tools, vertical and lateral forces acting on them, patterns of soil failure, displacement of soil particles, forces at the wheel-soil interface, wheel sinkage, rolling resistance, contact area of wheels, and soil stress at various depths *(Ani et al., 2018; Ghereș et al., 2013)*.

The resistance force to the movement of agricultural aggregates arises from the friction between the wheels and the soil surface, the friction of the axles in the wheel bearings, and the deformation of the soil by the aggregate's wheels *(Dobrescu, 1981)*. This force is influenced by the type and construction of the driving and supporting wheels, the weight of the aggregate, and the properties of the soil on which it operates. On loose, highly moist, or uneven soils, the resistance to movement is higher compared to operating on compacted, smooth-surfaced soils. Soil surface irregularities increase rolling resistance due to additional soil deformation, deviation of the machines in the aggregate from the forward direction, and the emergence of additional inertial forces *(Gill, Vanden Berg, 1968; Kheiralla et al., 2004)*.

Rolling resistance is also affected by the relaxation time of the soil and the tires after their mutual deformation at the point of contact. As relaxation time increases, soil deformation decreases, and vice versa (*Koolen et al., 1983*). Low-pressure tires, typical for agricultural machines, have a larger contact area with the soil, resulting in lower specific pressure and reduced soil deformation. Consequently, rolling resistance on deformable soil surfaces is lower, but the tires undergo more significant deformations, reducing their service life. On loose soils, aggregates equipped with low-pressure tires or tracks exhibit lower resistance to movement compared to those with metal wheels or high-pressure tires *(Bennett et al., 2019; Jun et al., 2024)*.

The traction resistance of agricultural machines, like other aggregate parameters, exhibits a pronounced random nature *(Махереș et al., 2013)*. Several factors influence the variability characteristics, the most important being soil type, soil compaction, moisture, non-uniformity, texture, and structure of the processed soil, as well as certain variations occurring at the drive wheels and articulation points of the aggregate. These factors have a more pronounced effect on aggregates with high-powered tractors and at high working speeds *(Abo et al., 2011; Biriș et al., 2017)*.

The variation in the working resistance of agricultural machines also leads to the tractor engine operating under unstable conditions. Depending on this variation and the engine load coefficient, power losses of up to 10-15% can occur. The load regime of the aggregate machines induces random dynamic processes in all aggregate components *(Dobrescu, 1981; Md-Tahir et al., 2021)*.

The random nature of the engine torque further affects the tractor's traction characteristics. A negative impact can be observed on the adhesion force and the slippage of the driving wheels. At equal traction forces, higher dispersion in their value leads to increased tractor slippage. Additionally, when the frequency of traction resistance variation is high, significant power losses occur due to slippage *(Md-Tahir et al., 2021; McKyes, 1985)*. The traction resistance value of agricultural machines considered in calculations is the average value obtained from statistical processing of traction diagrams recorded during operation *(Trendafilov et al., 2023; Varani et al., 2023)*. As the working speed increases, the working resistance of agricultural machines also increases. The rate of this increase depends on the type of working parts of the machine and the nature of the working process *(Cardei et al., 2023)*.

It is imperative to determine the optimal operating parameters for agricultural implements designed for seedbed preparation, ensuring full utilization of the tractor engine's power and operating the implement at maximum working capacity, thereby achieving an optimal power balance *(Legay, 1988; Vlăduț et al., 2018)*. Currently, this can be accomplished using analytical and numerical calculation algorithms, which can be implemented on modern agricultural tractors and are presented within this study *(Vlăduțoiu et al., 2017; Simionescu et al., 1995)*.

The main objective of the paper is to determine the optimal operational parameters of a plowing aggregate composed of a 180 HP tractor and a 5-moldboard plow, specifically: *v* - working speed and *B* - working width, which ensure the full utilization of the tractor engine's power and the use of the aggregate at its maximum working capacity. As will be further shown, the power balance equation can be used as an objective function for optimizing the power balance.

MATERIALS AND METHODS

The study presented in this paper was conducted for an aggregate consisting of a 180 HP tractor (A 1800-A) and a variable working width plough with 5 moldboards (PP5VM), designed to perform plowing operations on flat terrain or slopes with a maximum inclination of 6° (figure 1).

Fig. 1 - The 180 HP tractor and a plough with variable working width with 5 moldboards

The aggregate requires power for movement and power for performing the plowing operation. The tractor's drive wheels generate the power necessary to operate the aggregate. As a constraint, the coefficient of utilization of travel time, τ_d, can be considered, knowing that its level is proportional to the working capacity of the aggregate *(Dobrescu, 1981)*.

The power balance of a plowing aggregate can be expressed by the following equation:

$$P_M \cdot \tau_M = \frac{f_1 \cdot G \cdot v + f_v \cdot G \cdot v^2 + k \cdot a \cdot B \cdot v + \varepsilon \cdot a \cdot B \cdot v^3}{\eta_T \cdot (1 - \delta_p)} \tag{1}$$

where: P_M - is the power developed by the engine, (W); η_M - the engine power utilization coefficient, η_p - the efficiency of the transmission at the power take-off and the driving wheels; f_1 - proportionality coefficient; f_v - coefficient of resistance increase due to speed growth, (s/m); δ_p - tractor slippage; G - the weight of the aggregate, (N); v – the working speed (m/s); B – the working width, (m); a – the working depth, (m); k – the resistance to deformation and crumbling of the soil furrow, (N/m^2); ε - the coefficient characterizing the shape of the crown and the properties of the soil, (N·s^2/m^4).

The terms in the first fraction of equation (1) represent the power required for moving the aggregate, while the terms in the second fraction represent the power required for plowing. If the coefficient k is determined without considering the friction between the working parts and the furrow, then the last two terms correspond to Goriacichin's formula *(Cardei et al., 2023)*. These frictions are accounted for when determining the coefficients f_1 and f_v.

If it is considered that there is a relationship between the weight of the aggregate and the working width in the form of $G_B = \alpha \cdot B$, where α is the specific weight of the aggregate per working width, (N/m), the objective function becomes:

$$P_M \cdot \eta_M \cdot \eta_T \cdot (1 - \delta_p) = f_1 \cdot \alpha \cdot B \cdot v + f_v \cdot \alpha \cdot B \cdot v^2 + k \cdot a \cdot B \cdot v + \varepsilon \cdot a \cdot B \cdot v^3 \tag{2}$$

The optimal parameters B and v are those that ensure maximum productivity at the engine power level. Since the technical productivity of plowing aggregates is related to the level of the time utilization coefficient of travel, τ_d, this constitutes the constraint:

$$\frac{v_g \cdot L}{v_g \cdot L + v \cdot (\Omega + m_1 \cdot B)} - \tau_d = 0 \tag{3}$$

The optimal solution can be found using the Lagrange multiplier and the constraint *(Simionescu et al., 1995)*:

$$\begin{cases} \dfrac{\partial P_M}{\partial v} + \lambda \cdot \dfrac{\partial \tau_d}{\partial v} = 0 \\ \dfrac{\partial P_M}{\partial B} + \lambda \cdot \dfrac{\partial \tau_d}{\partial B} = 0 \end{cases} \tag{4}$$

and the constraint:

$$\frac{m}{m + v \cdot (\Omega + m_1 \cdot B)} - \tau_d = 0 \tag{5}$$

in which m and m_1 are operating parameters dependent on the idle travel speed of the aggregate (v_g), the plot length (L), the length of the path travelled without working (S_g), the plot width (C), ($m = v_g \cdot L$, $m_1 = S_g^s / C$).

The terms in the system of equations (4) are determined using *(Dobrescu, 1981)*:

$$\frac{\partial P_M}{\partial v} = f_1 \cdot \alpha \cdot B + 2 \cdot f_v \cdot \alpha \cdot B \cdot v + k \cdot a \cdot B + 3 \cdot \varepsilon \cdot a \cdot B \cdot v^2 \tag{6}$$

$$\frac{\partial P_M}{\partial B} = f_1 \cdot \alpha \cdot v + f_v \cdot \alpha \cdot v^2 + k \cdot a \cdot v + \varepsilon \cdot a \cdot v^3 \tag{7}$$

$$\frac{\partial \tau_d}{\partial v} = \frac{m \cdot (\Omega + m_1 \cdot B)}{[m + v \cdot (\Omega + m_1 \cdot B)]^2} \tag{8}$$

$$\frac{\partial \tau_d}{\partial B} = \frac{m_1 \cdot m \cdot v}{[m + v \cdot (\Omega + m_1 \cdot B)]^2} \tag{9}$$

A system of three equations with three unknowns is obtained:

$$\begin{cases} f_1 \cdot \alpha \cdot B + 2 \cdot f_v \cdot \alpha \cdot B \cdot v + k \cdot a \cdot B + 3 \cdot \varepsilon \cdot a \cdot B \cdot v^2 - \lambda \cdot \dfrac{m \cdot (\Omega + m_1 \cdot B)}{[m + v \cdot (\Omega + m_1 \cdot B)]^2} = 0 \\ f_1 \cdot \alpha \cdot v + f_v \cdot \alpha \cdot v^2 + k \cdot a \cdot v + \varepsilon \cdot a \cdot v^3 - \lambda \cdot \dfrac{m_1 \cdot m \cdot v}{[m + v \cdot (\Omega + m_1 \cdot B)]^2} = 0 \\ \dfrac{m}{m + v \cdot (\Omega + m_1 \cdot B)} - \tau_d = 0 \end{cases} \tag{10}$$

The variable λ is eliminated by equating its value from the first equation of the system (10) with the value obtained from the second equation, resulting in:

$$3\cdot v^2+2\cdot v\cdot\left[\frac{m\cdot(1-\tau_d)}{\tau_d\cdot\Omega}-\frac{\alpha\cdot f_v}{\varepsilon\cdot a}\right]+\frac{f_1\cdot(f_v\cdot\alpha+k\cdot a\cdot\Omega}{\varepsilon\cdot\alpha\cdot\Omega}=0 \tag{11}$$

from which the expression for the optimal speed is obtained:

$$v_{opt}=\frac{1}{3}\cdot\left[\frac{m\cdot(1-\tau_d)}{\tau_d\cdot\Omega}-\frac{\alpha\cdot f_v}{\varepsilon\cdot a}+\sqrt{\left(\frac{\alpha\cdot f_v}{\varepsilon\cdot a}\right)^2+\left(\frac{m\cdot(1-\tau_d)}{\tau_d\cdot\Omega}\right)^2+\frac{\alpha\cdot f_v\cdot m\cdot(1-\tau_d)}{\varepsilon\cdot a\cdot\tau_d\cdot\Omega}-\frac{3\cdot(\alpha\cdot f_1+k\cdot a)}{\varepsilon\cdot a}}\right] \tag{12}$$

From the third equation of the system (10), it results:

$$B_{opt}=\frac{m\cdot(1-\tau_d)-\tau_d\cdot\Omega\cdot v_{opt}}{\tau_d\cdot m_1\cdot v_{opt}} \tag{13}$$

The productivity of the aggregate is:

$$U_{\tau_d}=v_{opt}\cdot B_{opt}\cdot\tau_d \tag{14}$$

The power required to operate the aggregate is calculated using the following equation:

$$P_{min}=f_1\cdot\alpha\cdot B_{opt}\cdot v_{opt}+f_v\cdot v^2\cdot\alpha\cdot B_{opt}+B_{opt}\cdot v_{opt}\cdot a\cdot k+\varepsilon\cdot a\cdot B_{opt}\cdot v_{opt}^3 \tag{15}$$

The above calculation can be applied when studying the parameters of a new implement or when forecasting the development of plowing aggregates.

For existing aggregates, the relationship between the weight of the aggregate and its components is established using the following formula:

$$G=G_0+G_B=G_0+\alpha\cdot B \tag{16}$$

where: G_0 - represents the constant weight of the aggregate (the weight of the tractor and the carrying device), and α - is the specific weight of the part that changes with the working width B.

In this case, the mathematical model takes the form of:

$$P_M\cdot\eta_M\cdot\eta_T\cdot(1-\delta)=f_1\cdot G_0\cdot v+f_1\cdot\alpha\cdot B\cdot v+f_v\cdot G_0\cdot v^2+f_v\cdot\alpha\cdot B\cdot v^2+k\cdot a\cdot B\cdot v+\varepsilon\cdot a\cdot B\cdot v^3 \tag{17}$$

and:

$$\frac{m}{m+v\cdot(\Omega+m_1\cdot B)}-\tau_d=0 \tag{18}$$

By deriving the objective function and the constraints, the following is obtained:

$$\begin{cases}\dfrac{\partial P_M}{\partial v}=f_1\cdot(G_0+\alpha\cdot B)+2\cdot f_v\cdot v\cdot(G_0+\alpha\cdot B)+k\cdot a\cdot B+3\cdot\varepsilon\cdot a\cdot B\cdot v^2\\ \dfrac{\partial P_M}{\partial B}=f_1\cdot\alpha\cdot v+f_v\cdot\alpha\cdot v^2+k\cdot a\cdot v+\varepsilon\cdot a\cdot v^3\\ \dfrac{\partial\tau_d}{\partial v}=\dfrac{m\cdot(\Omega+m_1\cdot B)}{[m+v\cdot(\Omega+m_1\cdot B)]^2}\\ \dfrac{\partial\tau_d}{\partial B}=\dfrac{m_1\cdot m\cdot v}{[m+v\cdot(\Omega+m_1\cdot B)]^2}\end{cases} \tag{19}$$

A system of three equations with three unknowns is formulated using the coefficient λ:

$$\begin{cases}f_1\cdot G+f_1\cdot\alpha\cdot B+2\cdot f_v\cdot v\cdot G+2\cdot f_v\cdot\alpha\cdot B\cdot v+k\cdot a\cdot B+3\cdot a\cdot B\cdot v^2-\lambda\cdot\dfrac{m\cdot(\Omega+m_1\cdot B)}{[m+v\cdot(\Omega+m_1\cdot B)]^2}=0\\ f_1\cdot\alpha\cdot v+f_v\cdot\alpha\cdot v^2+k\cdot a\cdot v+\varepsilon\cdot a\cdot v^3-\lambda\cdot\dfrac{[m+v\cdot(\Omega+m_1\cdot B)]^2}{m_1\cdot m\cdot v}=0\\ \dfrac{m}{m+v\cdot(\Omega+m_1\cdot B)}-\tau_d=0\end{cases} \tag{20}$$

By eliminating the variable λ and performing the simplifications, the following is obtained:

$$f_1\cdot\alpha\cdot\Omega+f_v\cdot\alpha\cdot v\cdot\Omega+k\cdot a\cdot\Omega\cdot v^2-f_1\cdot G\cdot m_1-2\cdot f_v\cdot v\cdot\Omega\cdot B\cdot m_1-2\cdot\varepsilon\cdot a\cdot B\cdot v^2\cdot m_1=0 \tag{21}$$

If B is replaced in the last equation of the system (20), the result is:

$$3\cdot v^2+2\cdot v\cdot\left[\frac{f_v\cdot(\alpha\cdot\Omega-G\cdot m_1)}{\varepsilon\cdot a\cdot\Omega}-\frac{m\cdot(1-\tau_d)}{\tau_d\cdot\Omega}\right]+\frac{f_1\cdot(\alpha\cdot\Omega-G\cdot m_1+k\cdot a\cdot\Omega)}{\varepsilon\cdot a\cdot\Omega}-\frac{f_v\cdot\alpha\cdot m_1\cdot(1-\tau_d)}{\varepsilon\cdot a\cdot\tau_d\cdot\Omega}=0 \tag{22}$$

from which the expression for the optimal speed results:

$$v_{opt}=\frac{1}{3}\cdot\left[\frac{m\cdot(1-\tau_d)}{\tau_d\cdot\Omega}-\frac{f_v\cdot(\alpha\cdot\Omega-m_1\cdot G)}{\varepsilon\cdot a\cdot\Omega}+\sqrt{\left(\frac{m\cdot(1-\tau_d)}{\tau_d\cdot\Omega}\right)^2+\frac{f_v\cdot m\cdot(1-\tau_d)\cdot(\alpha\cdot\Omega+2\cdot m_1\cdot G)}{\varepsilon\cdot a\cdot\tau_d\cdot\Omega^2}\left(\frac{\alpha\cdot f_v}{\varepsilon\cdot a}\right)^2-\frac{3\cdot f_1\cdot(\alpha\cdot\Omega-m_1\cdot G)+3\cdot k\cdot a\cdot\Omega)}{\varepsilon\cdot a\cdot\Omega}}\right] \tag{23}$$

The optimal working width, B_{opt}, is determined by the relationship of the maximum working capacity of the aggregate:

$$B_{opt}=\frac{m\cdot(1-\tau_d)-\tau_d\cdot\Omega\cdot v_{opt}}{\tau_d\cdot m_1\cdot v_{opt}}\qquad\text{(m)} \tag{24}$$

If the maximum loading of the tractor is desired, the following relation (25) results:

$$B_{opt}=\frac{P_a\text{-}f_1\cdot G_0\cdot v_{opt}\text{-}f_v\cdot G_0\cdot v_{opt}^2}{f_1\cdot\alpha\cdot v_{opt}+f_v\cdot\alpha\cdot v_{opt}^2+k\cdot a\cdot v_{opt}+\varepsilon\cdot a\cdot v_{opt}^3}\quad \text{(m)} \tag{25}$$

If in equation (23) of the working speed, $G_B=\alpha\cdot B=0$ is assumed, equation (12) is obtained. If it is assumed that the weight of the entire aggregate is constant, i.e., $\alpha = 0$, the following results:

$$v_{opt}=\frac{1}{3}\cdot\left[\frac{m\cdot(1\text{-}\tau_d)}{\tau_d\cdot\Omega}+\frac{f_v\cdot m_1\cdot G}{\varepsilon\cdot a\cdot\Omega}+\sqrt{\left(\frac{m_1\cdot G\cdot f_v}{\varepsilon\cdot a\cdot\Omega}\right)^2+\left(\frac{m\cdot(1\text{-}\tau_d)}{\tau_d\cdot\Omega}\right)^2+\frac{2\cdot m_1\cdot G\cdot f_v\cdot m\cdot(1\text{-}\tau_d)}{\varepsilon\cdot a\cdot\tau_d\cdot\Omega^2}+\frac{3\cdot f_v\cdot m_1\cdot G\text{-}3\cdot k\cdot a\cdot\Omega}{\varepsilon\cdot a\cdot\Omega}}\right] \tag{26}$$

By applying equation (25) and the corresponding computational algorithm, a Python code was developed, allowing the calculation and plotting of the optimal values for the plow's total working width (figure 2.a), as well as the 3D generation of this dependency on the working speed and the utilization coefficient of the displacement time (τ_d) (figure 2.b).

```python
import numpy as np
import matplotlib.pyplot as plt

m_1 = 45
m = 700
omega = 40

tau_d_values = [0.7, 0.71, 0.72, 0.73, 0.74, 0.75, 0.76, 0.77, 0.78]

v_opt = np.linspace(0.5, 2.5, 100)

plt.figure(figsize=(8, 6))

for tau_d in tau_d_values:
    B_opt = (m * (1 - tau_d) - tau_d * omega * v_opt) / (tau_d * m_1 * v_opt)
    # Filtrare pentru valorile B_opt in intervalul [0,8, 2]
    mask = (B_opt >= 0.8) & (B_opt <= 2)
    plt.plot(v_opt[mask], B_opt[mask], label=f"$\\tau_d = {tau_d}$")

plt.title("Graph of $B_{opt}$ vs $v_{opt}$ for Different $\\tau_d$")
plt.xlabel("$v_{opt}$ (m/s)")
plt.ylabel("$B_{opt}$ (m)")
plt.ylim(0.8, 2)
plt.axhline(0, color='black', linewidth=0.8, linestyle='--')
plt.legend()
plt.grid()

plt.show()
```

a)

```python
%matplotlib notebook

import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

m_1 = 45
m = 700
omega = 40

v_opt_values = np.linspace(0.5, 2.5, 50)  # v_opt între 0.5 și 2.5
tau_d_values = np.linspace(0.7, 0.9, 50)    # tau_d între 0.7 și 0.9

V_OPT, TAU_D = np.meshgrid(v_opt_values, tau_d_values)

B_OPT = (m * (1 - TAU_D) - TAU_D * omega * V_OPT) / (TAU_D * m_1 * V_OPT)

mask = (B_OPT >= 1) & (B_OPT <= 2)

fig = plt.figure(figsize=(10, 7))
ax = fig.add_subplot(111, projection='3d')

sc = ax.scatter(V_OPT[mask], TAU_D[mask], B_OPT[mask], c=B_OPT[mask], cmap='viridis', marker='o')

cbar = fig.colorbar(sc, ax=ax, shrink=0.5, aspect=10)
cbar.set_label("$B_{opt}$ (m)")

ax.set_title("3D Plot of $B_{opt}$ as function of $v_{opt}$ and $\\tau_d$")
ax.set_xlabel("$v_{opt}$ (m/s)")
ax.set_ylabel("$\\tau_d$")
ax.set_zlabel("$B_{opt}$ (m)")

ax.zaxis.label.set_rotation(90)

ax.view_init(elev=30, azim=140)  # ROTIRE PENTRU VIZUALIZARE DIFERITĂ

plt.tight_layout()

plt.show()
```

b)

Fig. 2 – Python code for determining the optimal working width of a plow with 5 bodies

```python
import numpy as np
import matplotlib.pyplot as plt

G_pl = 30000  # N
f_pl = 0.35
k_0 = 50000   # N/m^2
eps = 1000    # N/m^3
B = 1.5

a_values = np.linspace(0.05, 0.3, 100)

v_pl_values_fixed = [1, 1.5, 2, 2.5]

plt.figure(figsize=(8, 6))

for v in v_pl_values_fixed:
    R_pl_values = G_pl * f_pl + k_0 * a_values * B + eps * a_values * B * (v ** 2)
    plt.plot(a_values, R_pl_values, label=f"v_pl = {v} m/s")

plt.title("Graph of $R_{pl}$ vs $a$ for Different Values of $v_{pl}$")
plt.xlabel("$a$ (m)")
plt.ylabel("$R_{pl}$ (N)")
plt.grid(True)
plt.legend()

plt.tight_layout()
plt.show()
```

a)

```python
import numpy as np
import matplotlib.pyplot as plt

G_pl = 30000  # N
f_pl = 0.35
k_0 = 50000   # N/m^2
eps = 1000    # N/m^3

B_values = [1.25, 1.35, 1.45, 1.55, 1.65]

a_values = np.linspace(0.05, 0.3, 100)

v_pl = 2

plt.figure(figsize=(8, 6))

for B in B_values:
    R_pl_values = G_pl * f_pl + k_0 * a_values * B + eps * a_values * B * (v_pl ** 2)
    plt.plot(a_values, R_pl_values, label=f"B = {B} m")

plt.title("Graph of $R_{pl}$ vs $a$ for Different Values of $B$")
plt.xlabel("$a$ (m)")
plt.ylabel("$R_{pl}$ (N)")
plt.grid(True)
plt.legend()

plt.tight_layout()
plt.show()
```

b)

Fig. 3 – Python code for determining the traction force of a plow with 5 plowshares

For the plowing work, the traction resistance force of the plow can be estimated using the following relation:

$$R_p = k_0 \cdot a \cdot B_l \quad \text{[N]} \tag{27}$$

in which: k_0 represents the specific soil resistance to plowing, (2 – 10)·10⁴ [N/m²]; a is the working depth [m]; B_l is the total working width of the plow [m].

In figure 3.a, a Python code is presented that allows the calculation and graphical plotting of the dependency of the plow's draft resistance on the working depth for different working speeds, and in figure 3.b, the same dependency is shown, but for various total working widths of a plow with 5 plowshares and a variable working width.

In figure 4.a, a Python code is presented that allows the calculation and 3D plotting of the plow's traction resistance as a function of working depth and working speed for a single soil type, while in figure 4.b, the same dependence is shown for four soil types with different specific resistances to tillage in the case of a plow with 5 plowshares and a variable working width.

```python
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

G_pl = 30000  # N
f_pl = 0.35
k_0 = 50000  # N/m^2
eps = 1000  # N/m^3
B = 1.5  # Constanta B (poți ajusta această valoare)

v_pl_values = np.linspace(0.5, 3.5, 100)  # Interval pentru v_pl
a_values = np.linspace(0.1, 0.3, 50)     # Interval pentru a

V_PL, A = np.meshgrid(v_pl_values, a_values)

R_pl_values = G_pl * f_pl + k_0 * A * B + eps * A * B * (V_PL)**2

fig = plt.figure(figsize=(10, 7))
ax = fig.add_subplot(111, projection='3d')

surf = ax.plot_surface(V_PL, A, R_pl_values, cmap='viridis')

cbar = fig.colorbar(surf, ax=ax, shrink=0.5, aspect=10)
cbar.set_label("$R_{pl}$ (N)")

ax.set_title("3D Surface Plot of $R_{pl}$ as function of $v_{pl}$ and $a$")
ax.set_xlabel("$v_{pl}$ (m/s)")
ax.set_ylabel("$a$ (m)")
ax.set_zlabel("$R_{pl}$ (N)")

plt.tight_layout()
plt.show()
```

a)

```python
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from matplotlib.lines import Line2D
import matplotlib.colors as colors

G_pl = 30000  # N
f_pl = 0.35
eps = 1000  # N/m^3
B = 1  # Constanta B

v_pl_values = np.linspace(0.5, 3.5, 100)
a_values = np.linspace(0.1, 0.3, 50)

V_PL, A = np.meshgrid(v_pl_values, a_values)

k_0_values = [30000, 50000, 70000, 90000]

all_R_pl_values = []
for k_0 in k_0_values:
    R_pl = G_pl * f_pl + k_0 * A * B + eps * A * B * (V_PL)**2
    all_R_pl_values.append(R_pl)

min_val = min(np.min(R) for R in all_R_pl_values)
max_val = max(np.max(R) for R in all_R_pl_values)

fig = plt.figure(figsize=(12, 9))
ax = fig.add_subplot(111, projection='3d')

norm = colors.Normalize(vmin=min_val, vmax=max_val)

cmap = plt.get_cmap('viridis')

surf_list = []
for i, k_0 in enumerate(k_0_values):
    R_pl_values = G_pl * f_pl + k_0 * A * B + eps * A * B * (V_PL)**2
    surf = ax.plot_surface(V_PL, A, R_pl_values, cmap=cmap, alpha=0.6, norm=norm)
    surf_list.append(surf)

custom_lines = [Line2D([0], [0], color=cmap(i / (len(k_0_values)-1)), lw=4) for i in range(len(k_0_values))]
ax.legend(custom_lines, [f"$k_0 = {k_0}$ N/m²" for k_0 in k_0_values], title="Values of $k_0$")

cbar = fig.colorbar(surf_list[0], ax=ax, shrink=0.5, aspect=10)
cbar.set_label("$R_{pl}$ (N)")

ax.set_title("3D Surface Plot of $R_{pl}$ as function of $v_{pl}$ and $a$ for different $k_0$")
ax.set_xlabel("$v_{pl}$ (m/s)")
ax.set_ylabel("$a$ (m)")
ax.set_zlabel("$R_{pl}$ (N)")

plt.tight_layout()
plt.show()
```

b)

Fig. 4 – Python code for determining the 3D variation of the draft force of a plow with 5 plowshares

The power required to pull the plow during the plowing process is calculated:

$$P_t = k_0 \cdot a \cdot B_l \cdot v_l \quad \text{[W]} \tag{28}$$

where v_l represents the working speed of the tillage aggregate [m/s].

The energy consumption per unit of processed surface is calculated using the following relation:

$$E_{ha} = \frac{P_t}{S_h} \quad \text{[Wh/ha]} \tag{29}$$

where S_h represents the working capacity of the plowing aggregate [ha/h].

In figure 5.a, a Python code is presented which, based on relation (29), allows for the calculation and graphical plotting of the dependence of energy consumption per unit of worked surface for a plow with 5 moldboards with variable working width, for different working depths according to the specific resistance to plowing, and in figure 5.b, the same dependence is calculated and plotted in 3D.

```python
import numpy as np
import matplotlib.pyplot as plt

B_l = 1.5
v_l = 2
W_h = 0.25

k_0_values = np.linspace(2e4, 10e4, 500)

a_values = [0.15, 0.175, 0.2, 0.215, 0.25, 0.275, 0.3]

plt.figure(figsize=(8, 6))

for a in a_values:
    P_t = k_0_values * a * B_l * v_l
    E_ha = P_t / W_h
    plt.plot(k_0_values, E_ha, label=f"a = {a}")

plt.title("Graph of $E_{ha}$ vs $k_0$ for Different $a$ values")
plt.xlabel("$k_0$ [N/m²]")
plt.ylabel("$E_{ha}$ [Wh/ha]")
plt.legend()

plt.grid(True)
plt.show()
```

a)

```python
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

B_l = 1.5
v_l = 2
W_h = 0.25

k_0_values = np.linspace(2e4, 10e4, 500)
a_values = np.linspace(0.15, 0.3, 6)

K0, A = np.meshgrid(k_0_values, a_values)

E_ha = (K0 * A * B_l * v_l) / W_h

fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')

surf = ax.plot_surface(K0, A, E_ha, cmap='viridis')

ax.set_title("Graph of $E_{ha}$ vs $k_0$ and $a$")
ax.set_xlabel("$k_0$ [N/m²]")
ax.set_ylabel("$a$ [m]")
ax.set_zlabel("$E_{ha}$ [Wh/ha]")

cbar = fig.colorbar(surf, ax=ax, shrink=0.5, aspect=5)
cbar.set_label("$E_{ha}$ [Wh/ha]")

plt.show()
```

b)

Fig. 5 – Python code for determining the variation in energy consumption per unit area worked for a plow with 5 moldboards

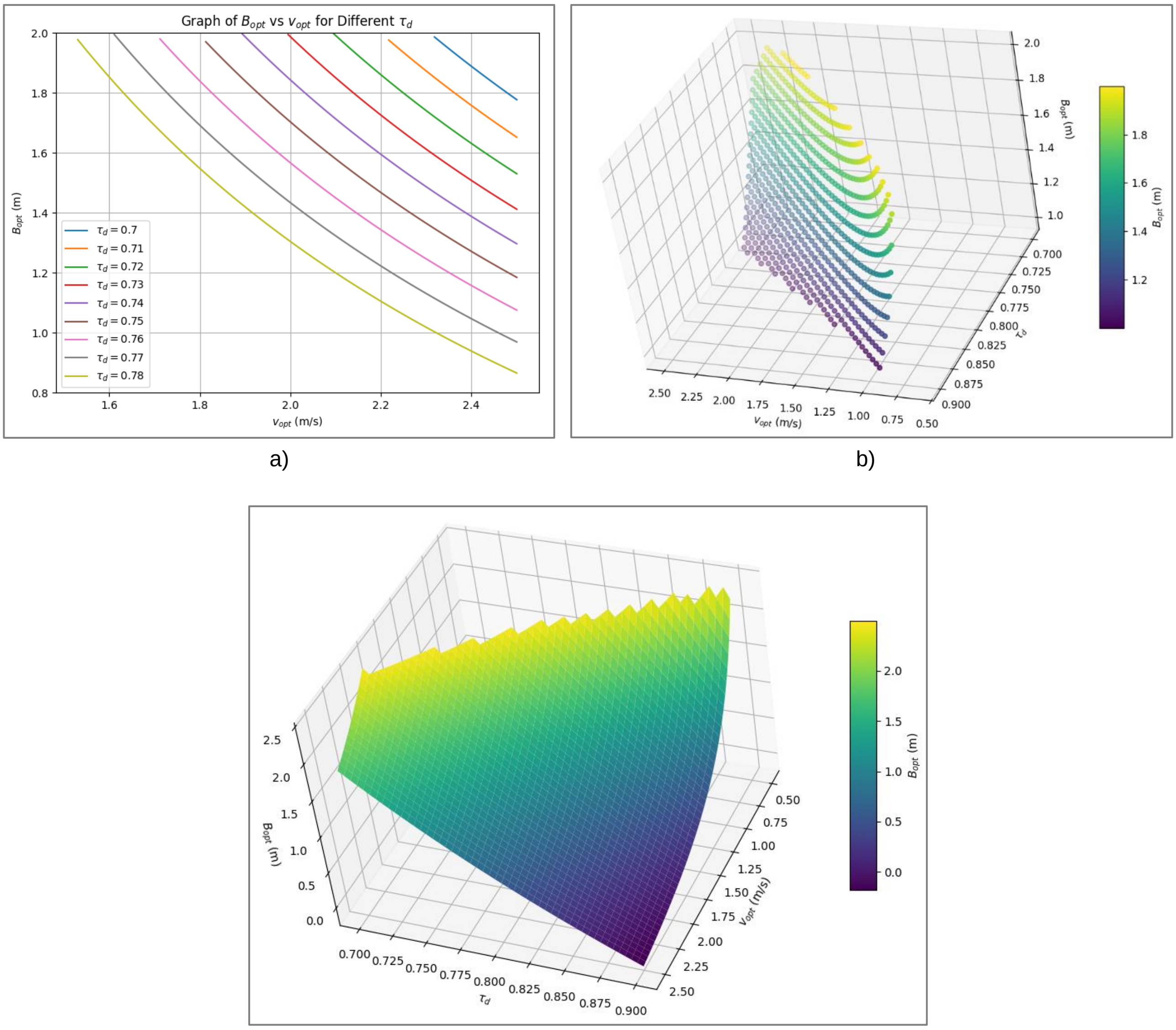
RESULTS

Through classical analytical calculation, applying the mathematical models embodied in equations (25) and (26), for a plowing unit operating under the conditions given in the first column of Table 1, the optimal parameters are obtained, which are found in the second column of the same table.

Table 1

The parameters resulting from the calculation for a plowing aggregate composed of a tractor A-1800 A and the PSP-5 (35) plow

The conditions of the plowing aggregate formed by the A-1800 A tractor and the PSP-5 (35) plow.	The parameters of the plowing aggregate resulting from the calculation
The weight of the tractor: G_t=97000 N *The weight of the plow:* G_p=30000 N *The weight of the aggregate:* $G=G_t+G_p$=127000 N τ_d=0.9; Ω=40; m_1=10; k=35000 N/m²; f_1=0.1; f_v=0.09; L=1000 m; v_g=0.5 m/s; a=0.25 m; ε=3000 N·s²/m⁴	v_{opt}=2.03 m/s B_{opt}=1.5 m

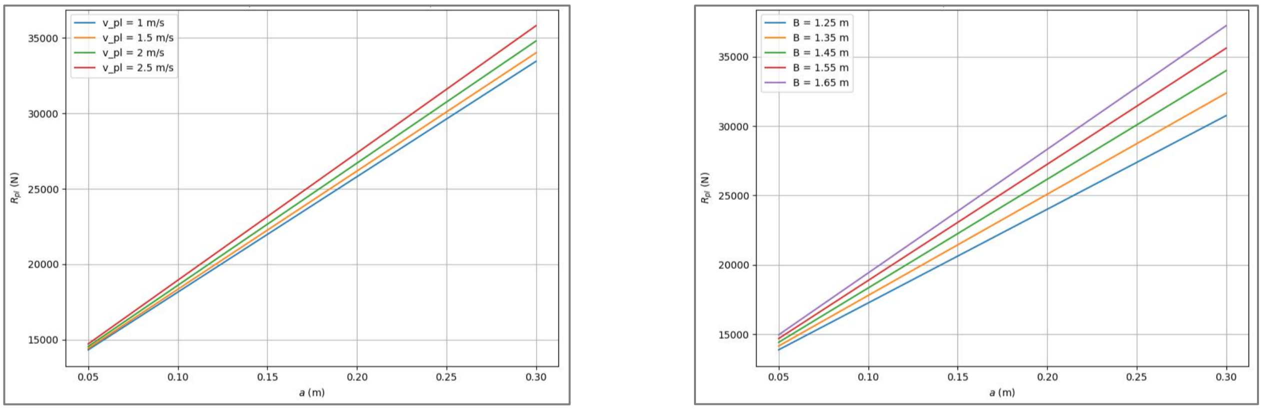


a) b)

c)

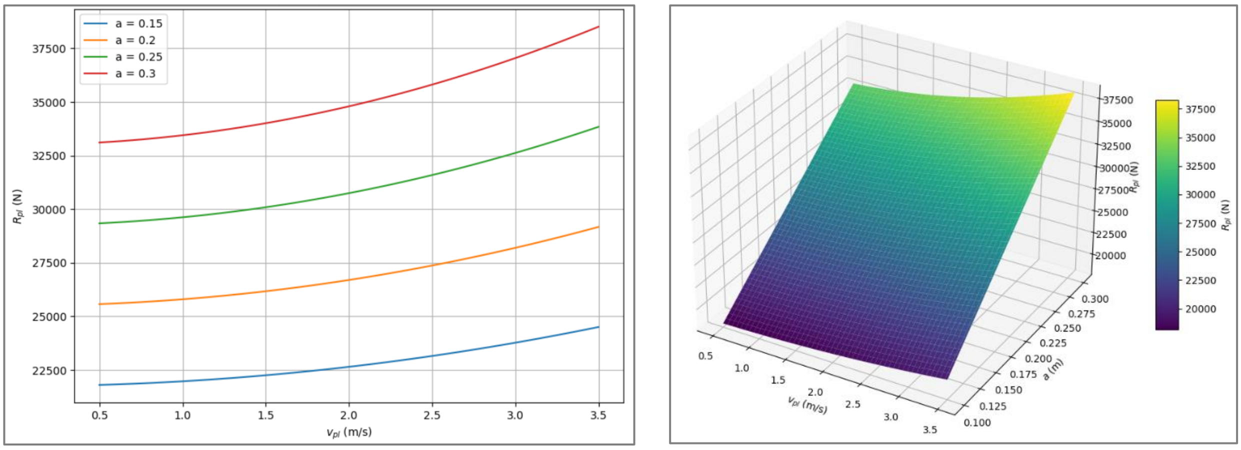
Fig. 6 – Graphical representation of the dependency of the optimal plowing width on the working speed and the travel time utilization coefficient for a plow with 5 moldboards

Using the calculation algorithms and Python codes presented in figure 2, the optimal values for the plow's total working width were calculated and plotted (figure 6.a), as well as the 3D variant of this dependency on working speed and the travel time utilization coefficient (τ_d) (figure 6.b). In addition, figure 6.c shows the 3D graph of the dependency of the optimal working width on the working speed - with values in the range of 0.5 - 2.5 m/s - and on the travel time utilization coefficient - with values in the range of 0.7–0.9.

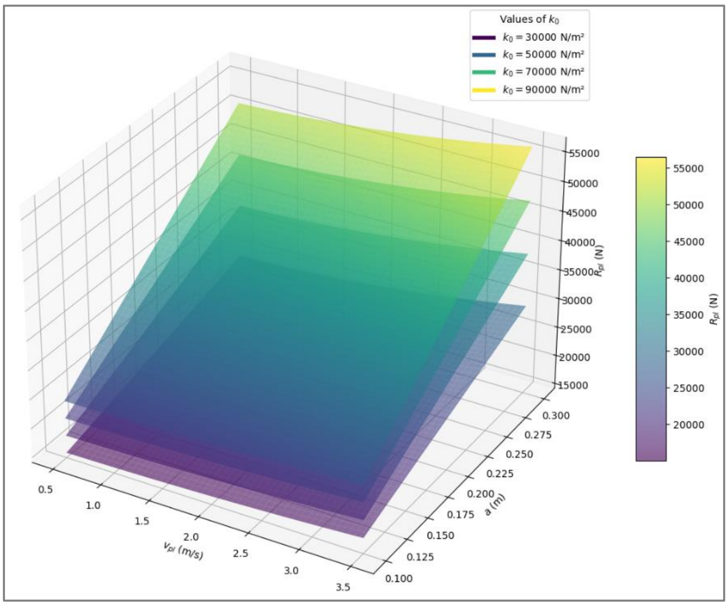


a) b)

Fig. 7 – Graphical representation of the dependence of the draft force of a plow with 5 moldboards at working depth for different working speeds and different total working widths



a) b)



c)

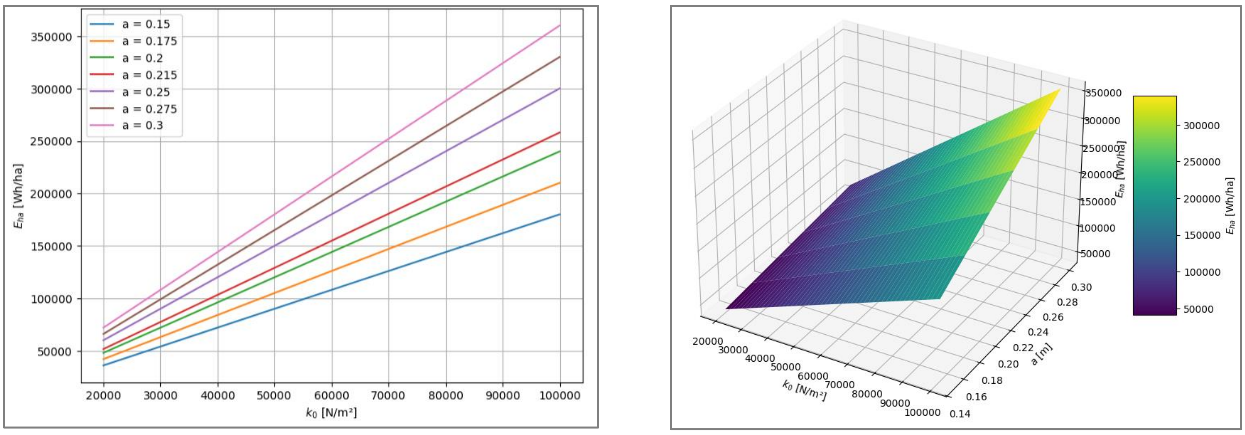
Fig. 8 – Graphical representation of the dependency of the traction force of a plow with 5 plowshares on working speed for various working depths and operating speeds

Using the calculation algorithms and Python codes presented in figure 3.a and figure 3.b, the dependence of the plow's draft resistance on working depth was calculated and plotted for different working speeds (figure 7.a), as well as the same dependence for various total working widths of a 5-furrow plow with variable working width (figure 7.b). It is easy to observe that the plow's draft resistance increases with the increase in working depth, for the different working speeds and various total working widths.

The draft force of the plow as its traveling speed increases, with the dependency $R_{pl}=f(v_{pl})$ being nonlinear (figure 8.a). The increase in draft force is smaller at lower speeds (0.5–2 m/s) and much more significant at higher working speeds (2–3.5 m/s).

In figure 8.b, the 3D graphical representation of the plows's draft force is shown, with working depth and working speed as the independent parameters, for a single type of soil. This was generated using the calculation algorithm and the Python program presented in figure 4.a. In figure 8.c, the same dependency is displayed, but for four different types of soil, using the calculation algorithm and Python code presented in figure 4.b.

Using the computational algorithms and the Python code presented in figure 5.a, the dependence based on relation (29) of the energy consumption per unit worked area for a plow with 5 moldboards of variable working width has been calculated and plotted, for different working depths as a function of the specific tillage resistance (different soil types) (figure 9.a). In figure 9.b, the 3D dependence of the energy consumption per unit worked area is presented as a function of the independent variables: working depth and specific tillage resistance, calculated and generated using the Python code presented in figure 5.b. The energy consumption per unit worked area for a plow with 5 moldboards of variable working width increases with an increasing working depth. The soil category and its physico-mechanical properties (specific tillage resistance) have a significant influence on the mechanical energy consumption.



a) b)

Fig. 9 – Graphical representation of the dependency of the traction force of a plow with 5 plowshares on working speed for different working depths and operating speeds

CONCLUSIONS

Optimal operation of the implements used for plowing is essential for efficient resource use and cost reduction. Fully utilizing the tractor's engine power and operating the plowing implement at its maximum capacity enables an optimal power balance. The optimal operating regime can be determined through numerical algorithms based on Lagrange multipliers, using computing programs (Python code in this case). To achieve maximum productivity at the engine power level, it is necessary to calculate the optimal working speed and optimal working width, depending on the coefficient of utilization of the plowing implement's transit time.

REFERENCES

[1] Abo Al-Kheer A., Kharmanda M.G., El Hami A, Mouazen A.M., (2011), Estimating the variability of tillage forces on a chisel plough shank by modeling the variability of tillage system parameters, *Computers and Electronics in Agriculture*, Volume 78, Issue 1, Pages 61-70. https://doi.org/10.1016/j.compag.2011.06.001

[2] Ani O. A., Uzoejinwa B. B., Ezeama A. O., Onwualu A. P., Ugwu S. N., Ohagwu C. J. (2018). Overview of soil-machine interaction studies in soil bins. *Soil and Tillage Research*, 175, pp. 13-27. https://doi.org/10.1016/j.still.2017.08.002

[3] Bennett J. M., Roberton S. D., Marchuk S., Woodhouse N. P., Antille D. L., Jensen T. A., Keller T., (2019), The soil structural cost of traffic from heavy machinery in Vertisols. *Soil and Tillage Research*, 185, pp.85-93. https://doi.org/10.1016/j.still.2018.09.007

[4] Biriş S. Şt., Ungureanu N., Gheorghiță N. E., Maican E., Vlăduț V., (2017), Theoretical research on enhancing the performance of cultivators working bodies, *Proceedings of the 45th International Symposium on Agricultural Engineering "Actual Tasks on Agricultural Engineering"*, pp.121-130, Opatija / Croatia; CABI Record Number: 20173126969

[5] Cardei P., Constantin N., Persu C., Muraru V., Sfiru R., Nițu M. (2023). Analysis of the dependence of traction resistance force on forward speed for tractor-cultivator agregates. *INMATEH-Agricultural Engineering*, 70(2). https://doi.org/10.35633/inmateh-70-57

[6] Croitoru Şt., Vladut V., Voicea I., Gheorghe Gh., Marin E., Vlăduţoiu L., Moise V., Boruz S., Pruteanu A., Andrei S., Păunescu D. (2017), Structural and kinematic analysis of the mechanism for arable deep soil loosening, *Proceedings of the 45th International Symposium on Agricultural Engineering "Actual Tasks on Agricultural Engineering"*, pp.207-216, Opatija / Croatia; CABI Record Number: 20173126979

[7] Dobrescu C. (1981). *Optimization of agricultural machinery parameters for the purpose of reducing consumption* (*Optimizarea parametrilor agregatelor agricole în scopul reducerii consumurilor*), Center for Teaching Materials and Agricultural Outreach, Bucharest, Romania

[8] Ghereș M.I., Ghereș M., (2013), Research on determination of interaction tools-soil, *ProEnviroment*, vol.6, pp. 262-270. http://journals.usamvcluj.ro/index.php/promediu

[9] Gill W.R., Vanden Berg G.E., (1968), Soil dynamics in tillage and traction, *U.S.A. Department of Agriculture*, Handbook 316, USA, Washington D.C.

[10] Hunt R.D. (1986). *Engineering Models for Agricultural Production*. *The Avi Publishing Co. Inc.*, Westport, Connecticut

[11] Jun Guo, Enhui Sun Yue Yang, Jun Lu. (2024). Soil stress analysis at different depth after agricultural vehicle operation. *INMATEH-Agricultural Engineering*, *73*(2). https://doi.org/10.35633/inmateh-73-12

[12] Kheiralla A.F., Yaha A., Zohadie M., Ishakt W. (2004). Modelling of power and energy requirements for tillage implements operating in Serdang sandy clay loam, Malaysia. *Soil & Tillage Research*. 78 (2004), pp. 21-34. DOI: 10.1016/j.still.2003.12.011

[13] Koolen A.J., Kuipers H., (1983), Agricultural soil mechanics, *Advanced Series in Agricultural Sciences*, Vol. 13. Springer, Heidelberg

[14] Legay I.M. (1988). *Methods and models in the study of complex systems. For a diversified agriculture* (*Methodes et models dans l'etude des sistemes complexes. Pour une agriculture divesifieé*), *L'Harmattan Publishing House*, Paris, France

[15] McKyes E., (1985), *Soil cutting and tillage*, Elsevier, Amsterdam-Oxford-New York-Tokyo

[16] Md-Tahir H., Zhang J., Xia J., Zhou Y., Zhou H., Du J., Mamona H. (2021). Experimental investigation of traction power transfer indices of farm-tractors for efficient energy utilization in soil tillage and cultivation operations. Agronomy, 11(1), 168. DOI: 10.3390/agronomy11010168

[17] Simionescu I., Dranga M., Moise V. (1995). *Numerical methods in engineering (Metode numerice în tehnică)*. Tehnică Publishing House, Bucharest, Romania

[18] Şandru A., Cristea I., Neculaiasa V., Popescu S. (1983). *Operation of agricultural machinery (Exploatarea utilajelor Agricole)*. Didactică şi Pedagogică Publishing House, Bucharest, Romania

[19] Trendafilov K., Tihanov G., Stoykova V., Shivacheva G. (2023). Algorithm for optimizing the movement of a mounted machine-tractor unit in the headland of an irregularly shaped field. *INMATEH-Agricultural Engineering*, *70*(2). https://doi.org/10.35633/inmateh-70-31

[20] Varani M., Mattetti M., Molari G., Biglia A., Comba, L. (2023). Correlation between power harrow energy demand and tilled soil aggregate dimensions. *Biosystems Engineering*, 225, pp. 54-68. DOI: 10.1016/j.biosystemseng.2022.11.008

[21] Vlăduț D.I., Biriș S-Șt., Vlăduț V., Cujbescu D., Ungureanu N., Găgeanu I., (2018), Experimental researches on the working process of a seedbed preparation equipment for heavy soils. *INMATEH - Agricultural Engineering*, Volume 55, Pages 27-34.

[22] Vlăduţoiu L., Cârdei P., Vlăduţ V., Fechete L., (2017), Modern trends in designing and selecting the machine/equipment for deep soil tillage, *16th International Scientific Conference "Engineering for Rural Development"*, pp.1415-1420, Jelgava / Latvia. DOI: 10.22616/ERDev2017.16.N320

[23] ***CIGR Handbook of Agricultural Engineering, Plant Production Engineering. Volume III. (1999), *American Society of Agricultural Engineers*.