RESEARCH ON THE OPTIMIZATION METHOD OF CLUTCH ENGAGEMENT TRAJECTORY IN THE TRACTOR POWER SHIFT PROCESS
/
拖拉机动力换挡过程离合器接合轨迹优化方法研究

Dongyang WANG[1)], Weipeng ZHANG[2)], Jiangtao JI[3)], Zuohua DONG[4)], Hongmei ZHU[1)], Zhanyu LI[1)]
[1)]College of Automotive and Transportation Engineering, Shenzhen Polytechnic University, Shenzhen/China;
[2)] National Key Laboratory of Agricultural Equipment Technology, China Academy of Agricultural Mechanization Science Group Co., Ltd, Beijing/ China
[3)] College of Agricultural Equipment Engineering, Henan University of Science and Technology, Luoyang/China
[4)] Changzhou Dongfeng Agricultural Machinery Group Co. Ltd., Changzhou/China
Tel: 86-15236286319; E-mail: zpt_wdy@szpu.edu.cn
DOI: https://doi.org/10.35633/inmateh-75-03

Keywords: *Clutch, Engagement trajectories, Pseudo-spectroscopy, Optimization*

ABSTRACT
To address the issue of reduced comfort and operational accuracy in tractors caused by the clutch engagement process in combination with automatic transmissions, dual-clutch transmissions, and hybrid power tractors, a new method is proposed that considers both the slipping process and synchronous instantaneous control at the moment of engagement. In this method, the optimal engagement process model of the clutch considering the clutch sliding process and synchronous instantaneous control was established, and the optimal trajectory of the clutch engagement process was solved based on the pseudo-spectral method. Then, the optimization results were compared with those obtained without considering the synchronous instantaneous. The results show that the proposed method considering the sliding process and synchronous instantaneous constraint can reduce the frictional loss of the clutch by 9%, and suppress the impact to below 10 m/s³. Finally, this method was applied to the control of tractor starting up, gear shifting and hybrid power mode switching processes. Simulation results demonstrate that this method can be effectively applied to these three operating conditions.

摘要
为解决搭配自动变速器、双离合器变速器和混合动力拖拉机离合器接合过程导致的拖拉机舒适性和作业精度降低问题，提出了一种考虑滑摩过程和同步瞬间控制的新方法。在该方法中，建立了考虑离合器滑摩过程和同步瞬间控制的离合器最优接合过程模型，并基于伪谱法求解了离合器接合过程的最优轨迹。结果表明，考虑滑动过程和同步瞬间约束优化方法可使离合器的摩擦损耗降低 9%，并可抑制冲击降低到 10 m/s³ 以下。最后，将该方法应用于拖拉机起步、换挡和混合动力模式切换过程的控制，仿真结果表明，该方法能较好地应用于上述三种工况。

INTRODUCTION
As a key component in the tractors' driveline, the clutch has important impact on the comfort of the vehicle during start-up, shifting and mode switching of hybrid vehicles (Fig.1) (*Park et al, 2021; Van Berkel et al, 2014; Lu, 2012; Li et al, 2020; Minh et al, 2012*). To ensure smooth driving and prolong the service life of components, the clutch is supposed to be engaged quickly and smoothly during the start-up, shift and mode switch process of tractors equipped with automatic transmission and hybrid-driven driveline (*Gavgani et al, 2016; Zhao et al. 2016; Fu et al., 2016*). Researches have shown that the fast and smooth engagement can be obtained by controlling the drive and driven disc of the clutch to engage in a certain trajectory (*Zhao. et al., 2016*). Therefore, it is necessary to optimize the engagement trajectory of the clutch.

[1] *Dongyang Wang, Lecture, Ph.D. Eng; Weipeng Zhang, Senior engineer Ph.D. Eng; Jiangtao Ji, Prof. Ph.D.Eng; Zuohua Dong, Senior engineer, M.S. Eng; Hongmei Zhu, Assoc. Prof. Ph.D. Eng; Zhanyu Li, Assoc. Prof. Ph.D. Eng.*

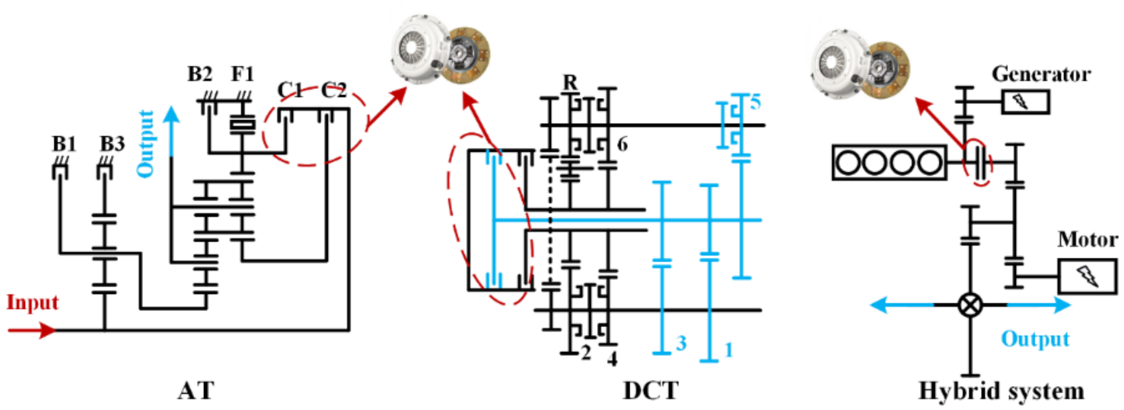

Fig. 1 - Typical driveline in the tractors

For the optimization of clutch engagement trajectory, many experts and scholars have carried out extensive and in-depth research. *Peng.et al., (2009)*, proposed a clutch engagement curve correction algorithm based on optimization theory, which corrects the original curve by judging the Hamilton function on the basis of the basic structure of the original curve. However, this method only analyzed the situation of the initial speed of the driven disc as 0, they did not analyze the gear shift process and the mode switch process of hybrid vehicles. To solve the problem of the variability of clutch engagement trajectory under uncertain factors such as system parameter perturbation and external load disturbance, *Li et. al., (2018),* proposed an optimization method which take the frictional loss and jerk as the optimization goal, and solve the optimization problem by using the Pontryagin theory. The result showed that, this method can reduce the frictional loss of the clutch during its engagement process, but the method did not solve the jerk of the synchronous instantaneous of the clutch. *Li et. al., (2018)*, optimized the shifting process with optimal control theory, and designed a linear quadratic optimal controller for the torque phase and the inertial phase. The results show that this method can reduce the jerk of torque phase and inertia phase during gear shifting process. To reduce the vibration during the inertial phase of the clutch engagement of shifting process, *Lu.et al., (2014)*, proposed to use the model predictive control method to control the inertial phase in the shifting process, and the results show that the method reduces the inertial phase vibration in the vehicle shifting process while ensuring the minimum change of the control amount in the shifting process. *Mesmer et al.* (*2017, 2018*), optimized the shifting process with the goal of minimizing the shifting time and frictional loss, and designed the optimal trajectory tracking controller for the shifting process by using the method of embedded nonlinear model predictive control, the simulation and experimental results show that the method can obtain better shifting quality, but the instantaneous synchronization of the clutch was not controlled in this study, resulting in residual oscillation on the drive shaft after clutch engagement. *Guo, (2017),* analyzed the clutch engagement process of hybrid vehicles from pure electric mode to hybrid drive state, he divided the clutch engagement process into two stages, slipping friction phased and synchronization phase. He proposed to use the method of Laguerre function set to optimize the control process of clutch engagement in hybrid vehicles, the results show that this method can reduce the torsional vibration of the transmission system during the engine starting process of hybrid vehicles.

Making the clutch engage fast and smoothly is an important measure to improve the vehicle start-up, shifting and hybrid vehicle mode switching process (*Wurm et al., 2016, Kim et al, 2017*), in view of the problems of synchronous instantaneous jerk existing in the current research, a method combining process control and terminal constraints was proposed.

The frictional loss and jerk during the clutch engagement process were chosen as the optimization goals during the slipping phase. In addition, the same acceleration of the drive and driven clutch was set as the terminal constraint. The pseudo-spectral method was used to solve this optimization problem, and the proposed method was applied to the start-up, shifting, and mode-switching processes of the hybrid vehicle.

MATERIALS AND METHODS

Modeling the clutch engagement process

In practice, the working state of the clutch is divided into two types, engagement and disengagement. During the engagement process, the actuator pushes the clutch driven disc move to the drive disc, and the disc rubs against each other to transmit the torque, which can be illustrated in Figure 2.

Assuming that the input torque is T_{in}, the torque transmitted during slipping phase is T_c, and the resistance clutch is T_r, according to Newton's second law, the clutch engagement process can be modeled (*Li, J., 2016*).

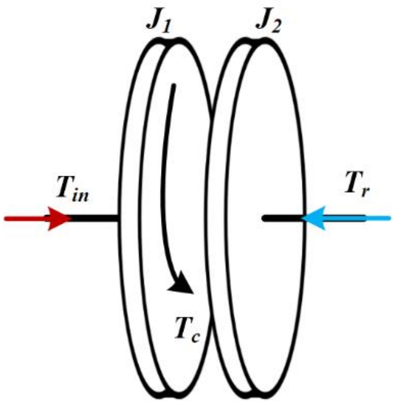

Fig. 2 - Model of clutch engagement process

$$\begin{cases} J_1 \dot{\omega}_1 = T_{in} - T_c \\ J_2 \dot{\omega}_2 = T_c - T_r \end{cases} \tag{1}$$

Take $x = [\omega_1, \omega_2, \omega_1 - \omega_2, T_{in}, T_c]^T$ as the state variable, $u = [\dot{T}_{in}, \dot{T}_c]^T$ as the control variable, the model of the clutch engagement process can be transferred into state space equation.

$$\begin{cases} \dot{x}_1 = \frac{1}{J_1}\left(x_4 - x_5\right) \\ \dot{x}_2 = \frac{1}{J_2}\left(x_5 - T_r\right) \\ \dot{x}_3 = \dot{x}_1 - \dot{x}_2 \\ \dot{x}_4 = u_1 \\ \dot{x}_5 = u_2 \end{cases} \tag{2}$$

where, J_1 is the inertia of drive disc of clutch, J_2 is the inertia of driven disc of clutch, ω_1 is the rotation speed of clutch drum and ω_2 is the rotation speed of clutch hub.

Indicators for clutch engagement process

1. Jerk. In the longitudinal dynamics of the tractor, the jerk (the change rate of longitudinal acceleration) is generally used to reflect the comfort of the tractor during acceleration. Moreover, the jerk can be represented by equation (3). When the value of the jerk is less than 10 m/s^3 during clutch engagement process, the tractor can get good comfort during acceleration (*Fu et al., 2016*).

$$j = \frac{da}{dt} = \frac{1}{J_e} \frac{d\left(T_c - T_r\right)}{dt} \tag{3}$$

To prevent the clutch from generating a large jerk during the engagement process, it is generally necessary to control change rate of the transmitted torque T_c when the clutch engages.

2. Frictional loss. The frictional loss of the clutch during the engagement process can be measured by the slipping work, it reflects the degree of the wear of the clutch during the engagement process. Therefore, it is necessary to control its slipping work during the clutch engagement process, and the clutch slipping work can be measured with equation (4).

$$W = \int_{t_s}^{t_e} T_c \left|\omega_1 - \omega_2\right| dt \tag{4}$$

Wherein, W is the clutch slipping work, t_s is the time when the clutch starts to slip with each other, and t_e is the time when the clutch engaged totally.

During clutch slipping phase, it is generally required that the frictional loss and jerk to be as small as possible. Meanwhile, it is important to control the jerk caused by the synchronous instantaneous. According to the above requirements, the optimization goal of the clutch engagement process can be established.

$$l=\int_{t_s}^{t_e}\lambda_1 u_1^2 dt+\lambda_2\int_{t_s}^{t_e}T_c\left|\omega_1-\omega_2\right|dt \tag{5}$$

where, λ_1, λ_2 is the weight coefficient, u_1 is the control variable. The objective function consists of two parts. The first part is the sum of squares of the torque change rate, which represents the limit of the torque change rate during the clutch engagement process; it also represents jerk control. The second part is expressed as the frictional loss during the clutch engagement process. In addition, keeping the acceleration of the two discs at the moment of synchronous can reduce the jerk generated by the clutch engagement synchronous instantaneous.

Based on the above objective function of the clutch engagement process, the clutch engagement process can be converted into an optimization problem, which can be expressed as equation (6).

$$\min J(x(t),u(t),t)=\int_{t_s}^{t_e}\lambda_1 u_1^2 dt+\int_{t_s}^{t_e}T_c\left|\omega_1-\omega_2\right|dt$$

$$st\begin{cases}\dot{x}=f\left(x,u,t\right)\\ x_1\left(t_e\right)-x_2\left(t_e\right)=0\\ \dot{x}_1\left(t_e\right)-\dot{x}_2\left(t_e\right)=0\\ T_{in\_\min}\le T_{in}\le T_{in\_\max}\\ T_{c\_\min}\le T_c\le T_{c\_\max}\\ x_{1\_\min}\le x_1\le x_{1\_\max}\\ x_{2\_\min}\le x_2\le x_{2\_\max}\end{cases} \tag{6}$$

wherein, T_{in_min} is the minimum of the input torque, the T_{in_max} is the maximum of the input torque, the T_{c_min} is the minimum torque transmitted during the clutch slipping process, the T_{c_max} is the maximum torque transmitted during the clutch slipping process, the x_{1_min} is the minimum speed of the clutch drum, the x_{1_max} is the maximum speed of the clutch drum, the x_{2_min} is the minimum speed of the clutch hub, and the x_{2_max} is the maximum speed of clutch hub.

It can be seen from the above optimization problem that this problem is an optimization problem with terminal constraints, especially for the clutch synchronous moment, the constraint problem of the acceleration of the main and slave disks. For optimization problems with terminal constraints, the general analytical method is difficult to solve, therefore, this paper uses the numerical solution method to solve.

When solving optimization problems with terminal constraints, pseudo-spectral method uses global interpolation polynomials to approximate state variables and control variables on a series of discrete points, and converts differential equation constraints into algebraic constraints by introducing pseudo-spectral difference matrices similar to finite difference matrices, and then solves nonlinear optimization problems with complex constraints. According to different distribution methods, pseudo-spectral methods can be divided into Legendre pseudo-spectroscopy, Radau pseudo-spectroscopy, Gauss pseudo-spectroscopy and Chebyshe pseudo-spectroscopy (*Feng.et al, 2023*).

Radau pseudo-spectroscopy (RPM) is a numerical solution algorithm for solving nonlinear optimization problems. The basic idea is to discretize first and then optimize. Firstly, the unknown state quantity and control quantity to be solved are discretized at a series of Legendre-Gauss-Radau (LGR) points, and then the state variables and control variables are approximated by constructing global interpolation polynomials, and the original kinetic differential equation is replaced by derivation of the state variables, the optimization problem of continuous system is discretized into a series of algebraically constrained nonlinear optimization problems, and the nonlinear optimization problem of algebraic constraint is solved by numerical solving algorithm.

Radau pseudo-spectral optimization problems can be divided into the following steps (*Feng et al, 2022*).

1) Time domain transformation:

To satisfy the time domain of the Legendre orthogonal polynomial, it is necessary to convert the time domain in the optimization problem to the standard time domain [-1,1], and the variable τ represents the transformed time variable.

$$\tau=\frac{2t-\left(t_f+t_0\right)}{t_f-t_0} \tag{7}$$

2) Polynomial approximation of state variables and control variables

Based on the normalized time variable, the Lagrange interpolation polynomial is used to approximate the state and control variable.

$$\begin{cases} \boldsymbol{x}(\tau)\approx X(\tau)=\sum_{j=1}^{N+1}X_jL_j(\tau) \\ \boldsymbol{u}(\tau)\approx U(\tau)=\sum_{j=1}^{N+1}U_jL_j(\tau) \end{cases} \tag{8}$$

wherein, $L_j(\tau)$ is the Lagrange interpolated polynomial.

$$L_j(\tau)=\prod_{\substack{i=1 \\ i\neq j}}^{N+1}\frac{\tau-\tau_i}{\tau_j-\tau_i}, \qquad j=1,\dots,N+1 \tag{9}$$

wherein, $\tau_1,\dots,\tau_N$ is the root of the Legendre polynomial, and τ_{N+1} is the unconfigured node, which represents the end time.

3) Polynomial approximation of differential equations

Based on the approximation of the state quantity, the approximate state variable can be derived.

$$\frac{dX}{dt}=\frac{t_f-t_0}{2}f\left(\boldsymbol{X}(\tau),\boldsymbol{U}(\tau),\tau\right) \tag{10}$$

$$\frac{dX}{d\tau}\equiv\dot{X}(\tau)=\sum_{j=1}^{k+1}X_j\dot{L}_j(\tau) \tag{11}$$

Substituting Equation 11 into 10 and discretizing at the LGR point, equation (12) is obtained.

$$\sum_{j=1}^{N+1}X_jD_{ij}-\frac{t_f-t_0}{2}f\left(\boldsymbol{X}(\tau),\boldsymbol{U}(\tau),\tau\right)=0 \tag{12}$$

where D_{ij} is the differential matrix of the Radau pseudo-spectral method, and it can be expressed by equation (13).

$$D_{ij}=\begin{cases} \dfrac{\dot{h}(\tau_i)}{\left(\tau_i-\tau_j\right)\dot{h}(\tau_i)} & i\neq j \\ \dfrac{\ddot{h}(\tau_i)}{2\dot{h}(\tau_i)} & i=j \end{cases} \tag{13}$$

After the conversion of the above steps, the optimization problem of the clutch engagement process can be expressed by equation (14).

$$\begin{aligned} &\min J=\int_{-1}^{1}l\left(x(\tau),u(\tau)\right)d\tau \\ &s.t.\begin{cases} \sum_{j=1}^{N+1}X_jD_{ij}-\dfrac{t_f-t_0}{2}f\left(X(\tau),U(\tau),\tau\right)=0 \\ C_{eq}\left(x(\tau),u(\tau)\right)=0 \\ C_{ieq}\left(x(\tau),u(\tau)\right)<0 \\ \phi\left(x(\tau_0),u(\tau_0),x(\tau_f),u(\tau_f),\tau_0,\tau_f\right)=0 \end{cases} \end{aligned} \tag{14}$$

where, $l(x(\tau), u(\tau))$ is the stage value function in the clutch engagement process. C_{eq} is the equality constraint, C_{ieq} is the inequality constraint, and ϕ is the terminal constraint.

Thus, the optimization problem of continuous system is transformed into a series of nonlinear programming problems with algebraic constraints, and the discretized algebraic optimization problem is solved by using the sequence quadratic programming (SQP).

RESULTS

To test the effectiveness of the algorithm. The Optimization results were compared with the results obtained without considering the terminal constraint case.

Case 1. With terminal constraints

In this case, the engagement of the clutch for start-up was optimized. The initial speed of the clutch hub was set to be 0 r/min, the speed of clutch drum was set to be 764 r/min. The optimization problem was solved with *GPOPS*. The optimized engagement trajectory is shown in the Figure 3 illustrating the rotation speed of the clutch drum and hub. It can be seen from this figure that the rotation speed of the clutch hub increases gradually. The rotation speed of the clutch drum is kept as low as possible to keep the speed difference between clutch drum and hub at a small value because the objective function requires the frictional loss as small as possible. The clutch drum has small fluctuation at the synchronization stage to make the acceleration of the drive and driven disc tend to be consistent. When synchronized, the acceleration of the drive and driven disc is the same, as shown in the enlarged section in Figure 3a.

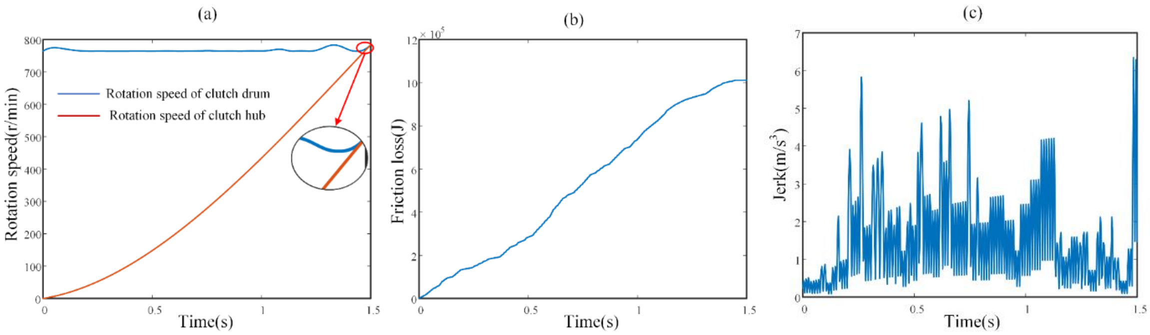

Fig. 3 - Clutch engagement trajectory optimized by pseudo-spectroscopy

Figure 3b shows the frictional loss during the engagement process. It can be seen from Figure 3b that the frictional loss is 10.12×10^5J. Figure 3c shows the value of the jerk during the engagement process, and it can be seen from Figure 3c that the value of the jerk is less than 10 m/s^3 during the entire engagement process of the clutch, and at the moment of clutch synchronization, although the value of the jerk is larger, it is still less than 10 m/s^3.

2. Case 2. Without terminal constraints

To check the effectiveness of the optimal problem that considers the process constraints and terminal constraints, the optimal problem without terminal constraints, which is mostly used in the optimal problem for clutch engagement process was also solved.

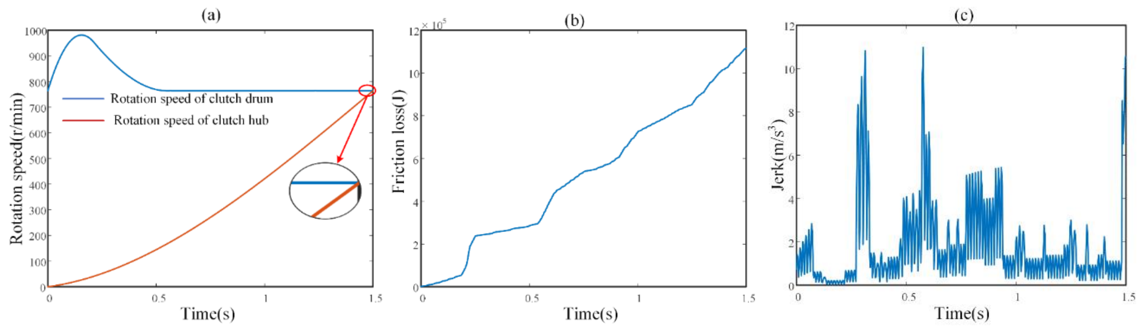

Fig.4 - Optimization results of clutch engagement process without considering terminal acceleration constraints

Figure 4 shows the results without considering the terminal constraint, Figure 4a is the rotation speed of the clutch drum and hub. It can be seen from the figure that the rotation speed of the clutch drum increases at first and then quickly decreases to the lowest value of the engine to make difference of the clutch drum and hub to be smallest during the whole engagement process. The rotation speed of the clutch drum was not adjusted at the synchronization stage, so the acceleration of the clutch drum and hub was inconsistent, as shown in the enlarged section of the figure. Figure 4b shows the frictional loss during clutch engagement, and it can be seen from the figure that the frictional work is 11.1×10^5J, which is higher than the values produced by the pseudo-spectroscopy. Figure 4c shows the value of the jerk during the engagement process. It can be seen from the figure that the maximum values of jerk exceed 10 m/s^3 at 0.25 s, 0.6 s and synchronization moment. Compared with the optimization without considering terminal constraints, the frictional loss is reduced by 9%, and the jerk is less than 10 m/s^3.

From the result of optimization with terminal constraints and without terminal constraints, it can be seen that the rotation speed of the clutch hub gradually increases, and the rotation speed of the clutch drum has been running at a lower value to keep the difference between the clutch drum and hub at a smaller value, so as to minimize the slip loss. Wherever, the rotation speed of the clutch drum was adjusted, and during the adjustment period, it resulted in two large jerks (0.25 s,0.6 s) during the engagement process.

Meanwhile, the rate of change of the torque transmitted by the clutch is constrained in the value function, resulting in a smaller value of the jerk throughout the clutch engagement process. When the terminal constraint is considered, the process constraint and the terminal constraint are both applied, resulting in a smaller jerk during the clutch engagement process, while if only the process constraint is applied without the terminal constraint, the clutch not only experiences a larger jerk at the slip phase, but also at the moment of synchronization.

Clutch engagement trajectory optimization under different operating conditions

(1) Start-up process

In vehicles equipped with automatic transmissions, the clutch driven disc starts at zero and the rotation speed of drive disc is limited by the minimum engine speed. Therefore, the cultch engagement process during the start-up process is optimized with the condition that the initial speed of the driven disc is zero.

Figure 5 shows the optimization curve of the clutch engagement process in the vehicle start-up process. To validate different start-up situations that are often present, three different start-up times was set to be 1.5 s, 2.0 s and 2.5 s, respectively. The optimization result is shown in Figure 5. It can be seen from the figure that the rotation speed curve of the driven disc is a smooth curve, and when the clutch is about to synchronize, the acceleration of the drive and driven disc is consistent, thus ensuring that the clutch has a small jerk during the slipping process and the synchronization moment.

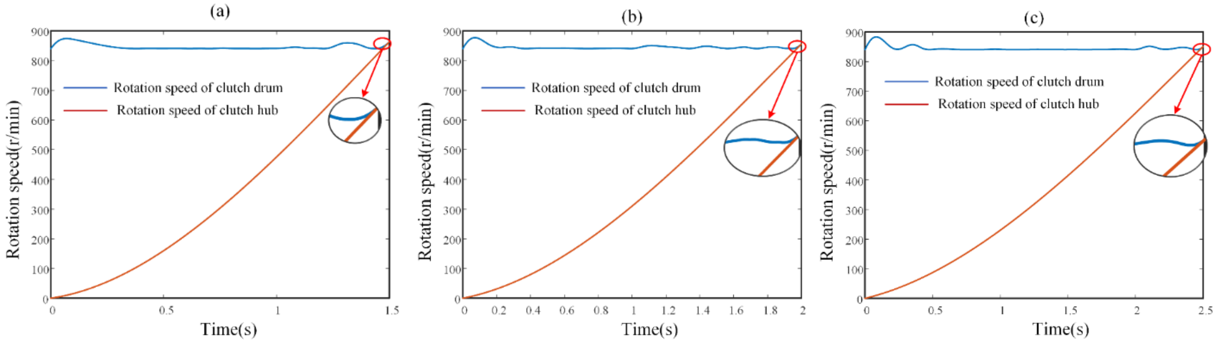

Fig. 5 - Start-up process, clutch engagement process optimized

Shift process

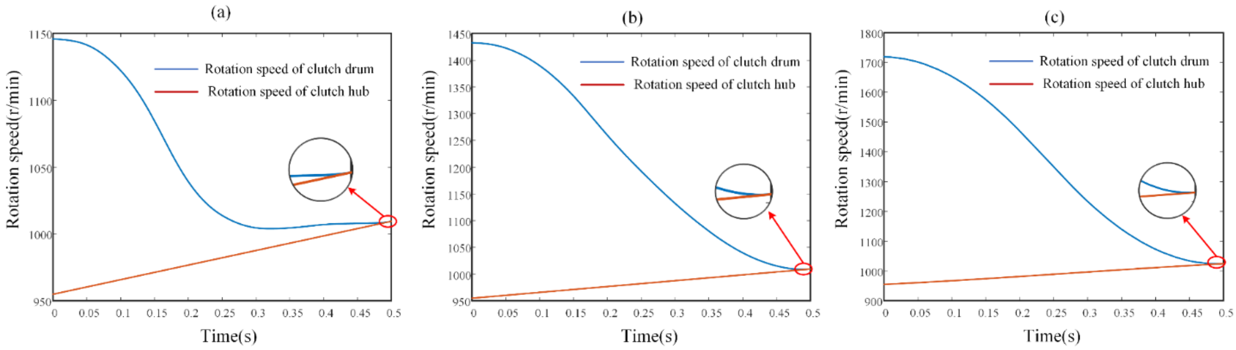

Fig. 6 - Optimal engagement curve of the clutch during the upshift process

Unlike in the vehicle starting process, the initial speed value of the clutch driven disc is not zero during the shifting process. In this study, the upshift process was taken as an example. During the upshift process, the initial speed value of the driven disc is not zero, and the speed of drive disc is higher than the speed of driven disc of the target gear.

To simulate the upshift process of different gears, the shift process time is set to 0.5 s, and three different gear values are set, the corresponding gear ratios are 1.2, 1.5, 1.8, respectively and the clutch driven disc speed value is set to 955 r/min. The optimization process of the optimal engagement path of the clutch in the upshift process under different transmission ratios is shown in Figure 6. From the figure, it can be seen that the drive disc speed decreases actively, the driven disc speed rises gradually, and at the end of the shift, the rotation speed of the drive and driven disc were consistent, and the angular acceleration value is the same at the same time.

Mode switching situation in the hybrid tractor

In parallel hybrid tractors, at high speeds, the engine is usually used to drive the vehicle directly to reduce the overall energy consumption. Parallel hybrid vehicles require clutch control when the engine intervenes to prevent sudden engine intervention and vibrations in the driveline. For the parallel hybrid vehicle, a small generator is used to drive the engine to start when the parallel hybrid vehicle engine is involved, and the speed of the engine is adjusted to be slightly higher than the speed of the clutch driven disc, meanwhile, the difference between the speed of drive and driven disc is small so that the clutch can be engaged smoothly and quickly when engaged. To verify the applicability of the method in the clutch engagement process of mode switch process, three different small speed differences were set to be 48 r/min (5 rad/s), 95 r/min (10 rad/s) and 144 r/min (15 rad/s), respectively, and the engagement time of the clutch was set to be 0.5 s, the optimization results are shown in Figure 7. It can be seen from Figure 7 (a) (b) that the rotation speed of driven disc increases gradually, the speed of the drive disc decreases gradually, and there are some fluctuations when the speed of the drive disc decreases, the reason for this fluctuation that the difference between the drive and driven disc is small, is that the engine needed to be adjusted to make the acceleration of the drive and driven disc be consistent. For the large value of the difference between the drive and driven disc, the engine speed can be smoothly reduced, which can be seen in Figure 7c, and the acceleration of drive and driven disc can still be consistent at the synchronization moment.

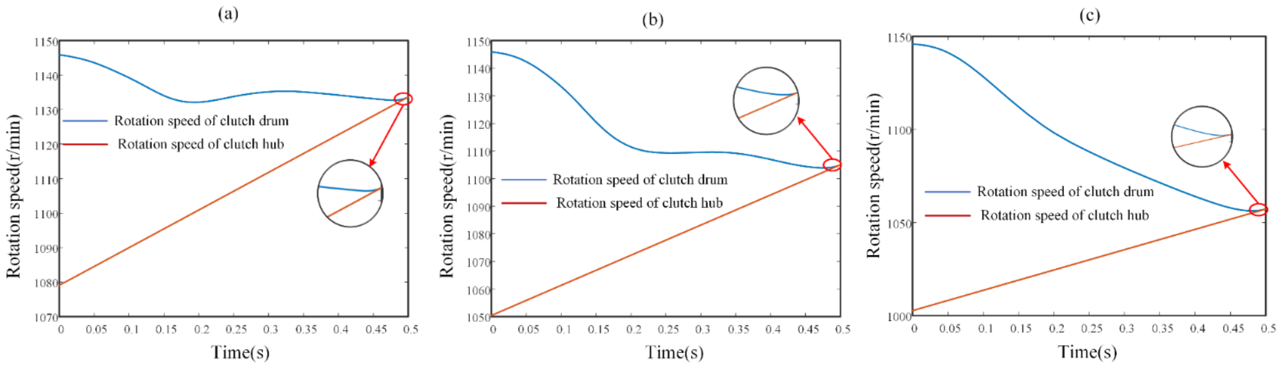

Fig. 7 - In hybrid systems, the clutch engages the optimal curve when the engine is involved

CONCLUSIONS

In this study, an optimization method that considers the slipping phase and synchronization moment of the clutch engagement was proposed. The frictional loss and jerk during the slipping phase were considered and the jerk at the moment of clutch synchronization was also considered as well, and the clutch engagement process was transformed into an optimization problem with terminal constraints. To solve this problem, the pseudo-spectral method was used and the optimization result was compared with the optimization method without considering the terminal constraints, and the results show that the frictional loss can be reduced by 9% and the jerk is lower than 10 m/s^3 by using the pseudo-spectral method considering the terminal constraint method, and this method was applied to the start-up, shift and mode switch process of the hybrid driveline. The results show that the method can ensure that the acceleration value of the clutch drum and hub tends to be consistent when the clutch is synchronized, which will reduce the residual oscillation on the drive shaft after clutch engagement (*Petrescu et al, 2018*).

This study optimizes the engagement trajectory of the clutch engagement process in terms of jerk and frictional loss, and solves the problem of jerk at the moment of clutch synchronization. However, the efficiency and energy consumption of the engine/motor connected to the clutch drum during its adjustment in the clutch engagement process have not been considered. In future work, the overall energy consumption of the process will be thoroughly analyzed. Additionally, a test rig will be developed to validate this method, and a novel control approach for tracking the optimal trajectory will also be explored.

ACKNOWLEDGEMENT

This work is supported by the National Key R&D Program Project of China (No.2023YFD2001102), Shenzhen Polytechnic University Research Fund (No. 6023310012K), Scientific Research Project of Shenzhen Polytechnic University（No. 6021310021K）.

REFERENCES

[1] Feng, J., Qin, D., Liu, Y., Wang, X., & Lv, H. (2023). Pseudo-spectral optimization and model predictive control of vehicle shift process with dual clutch transmission. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, *237*(16), 3789-3806. https://doi.org/10.1177/09544062221148040

[2] Feng, J., Qin, D., Liu, Y., Wang, X., & Lv, H. (2022). Pseudo-spectral optimization and data-driven control of vehicle start process with dual-clutch transmission. *Mechanism and Machine Theory*, *172*, 104814. https://doi.org/10.1016/j.mechmachtheory.2022.104814

[3] Fu, Y., Liu, Y., Cui, L., & Xu, X. (2016). Dynamic analysis and control strategy of wet clutches during torque phase of gear shift. *Journal of Mechanical Science and Technology*, *30*, 1479-1496.

[4] Gavgani, A. M., Sorniotti, A., Doherty, J., & Cavallino, C. (2016). Optimal gearshift control for a novel hybrid electric drivetrain. *Mechanism and Machine Theory*, *105*, 352-368. https://doi.org/10.1016/j.mechmachtheory.2016.06.016

[5] Guo D. (2017). *Torque-related Control Strategies for Mode Transition and Gear Shifting in Hevs (混合电动汽车模式切换及换档的转矩控制策略研究)*. Wuhan University of Technology, Wuhan.

[6] Kim, S., Oh, J., & Choi, S. (2017). Gear shift control of a dual-clutch transmission using optimal control allocation. *Mechanism and Machine Theory*, *113*, 109-125. https://doi.org/10.1016/j.mechmachtheory.2017.02.013

[7] Lu, T., Dai, F., Zhang, J., & Wu, M. (2012). Optimal control of dry clutch engagement based on the driver's starting intentions. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, *226*(8), 1048-1057. https://doi.org/10.1177/0954407011435465

[8] Li, J., Wang, Y., & Wang, Y. (2016). Optimal control for jitter suppression of clutch engagement in upshift process of electric vehicles. *Advances in Mechanical Engineering*, *8*(12), 1687814016674847. https://doi.org/10.1177/1687814016674847

[9] Li X.X., Wang A.L., Fan, X.C., Wu J. (2020). Optimization and tracking control of the clutch engagement process under non-stationary random cyclic conditions (非平稳随机循环工况离合器接合优化及跟踪控制). *Journal of Harbin Institute of Technology*, 52(7):52-58. DOI: 10.11918/201906001

[10] Li, C., Meng, F., Xi, J., & Zhai, Y. (2019). Starting shift control of heavy-duty automatic transmissions based on the optimal trajectory of turbine speed. *Mechanical Systems and Signal Processing*, *126*, 490-504. https://doi.org/10.1016/j.ymssp.2019.02.030

[11] Li, G., & Görges, D. (2018). Optimal control of the gear shifting process for shift smoothness in dual-clutch transmissions. *Mechanical Systems and Signal Processing*, *103*, 23-38. https://doi.org/10.1016/j.ymssp.2017.09.040

[12] Lu, X., Chen, H., Gao, B., Zhang, Z., & Jin, W. (2014). Data-driven predictive gearshift control for dual-clutch transmissions and FPGA implementation. *IEEE Transactions on Industrial Electronics*, *62*(1), 599-610. DOI: 10.1109/TIE.2014.2312312

[13] Minh, V.T., Mohd Hashim, F.B., & Awang, M. (2012). Development of a real-time clutch transition strategy for a parallel hybrid electric vehicle. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, *226*(2), 188-203. https://doi.org/10.1177/09596518114147

[14] Mesmer, F., Hentzelt, S., Szabo, T., & Graichen, K. (2017). Optimal control of dual-clutch transmissions with multiple groups. In *2017 IEEE International Conference on Advanced Intelligent Mechatronics (AIM)* (pp. 1749-1754). IEEE. DOI: 10.1109/AIM.2017.8014271

[15] Mesmer, F., Szabo, T., & Graichen, K. (2018). Embedded nonlinear model predictive control of dual-clutch transmissions with multiple groups on a shrinking horizon. *IEEE Transactions on Control Systems Technology*, *27*(5), 2156-2168. DOI: 10.1109/TCST.2018.2856191

[16] Park, J., & Choi, S. (2021). Optimization method of reference slip speed in clutch slip engagement in vehicle powertrain. *International Journal of Automotive Technology*, *22*, 55-67. https://doi.org/10.1007/s12239-021-0007-5

[17] Peng, J.X., Jin, H., Chen, H.Y. (2009). Engagement Curve of Clutch Based on Optimization Theory. *Transactions of Beijin institute of Technology*, (11), 964-968.

[18] Petrescu, R. V., Aversa, R., Apicella, A., & Petrescu, F. I. (2018). Dynamic Synthesis of a Dual-Clutch Automatic Gearboxes. *American Journal of Engineering and Applied Sciences*, *11*(2), 663-679.

[19] Van Berkel, K., Hofman, T., Serrarens, A., & Steinbuch, M. (2014). Fast and smooth clutch engagement control for dual-clutch transmissions. *Control Engineering Practice*, *22*, 57-68. https://doi.org/10.1016/j.conengprac.2013.09.010

[20] Wurm, A., & Bestle, D. (2016). Robust design optimization for improving automotive shift quality. *Optimization and Engineering*, *17*, 421-436.

[21] Zhao, Z., He, L., Zheng, Z., Yang, Y., & Wu, C. (2016). Self-adaptive optimal control of dry dual clutch transmission (DCT) during starting process. *Mechanical Systems and Signal Processing*, *68*, 504-522. https://doi.org/10.1016/j.ymssp.2015.06.012