THE FUNCTIONAL CONTROLABILITY OF MILK EJECTION OF THE ADAPTIVE MILKING SYSTEM

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ФУНКЦІОНАЛЬНА КЕРОВАНІСТЬ МОЛОКОВІДДАЧЕЮ АДАПТИВНОЇ ДОЇЛЬНОЇ СИСТЕМИ

Dmytriv I.V.[*1]
[1)] Lviv Polytechnic National University, Institute of Engineering Mechanics and Transport, Lviv / Ukraine
E-mail: Dmytriv_V@ukr.net

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ABSTRACT

The concept of functional controllability of the milk ejection is considered, which makes it possible to predict the intensity of milk ejection in the online mode of the milking machine. The architecture of the functional controllability by intensity of milk ejection is developed. Input and output parameters of the structural-functional scheme of adaptive control of milk ejection intensity are described. An analytical model of milk ejection intensity based on Pearson's distribution is developed. The milk ejection intensity for different productivity and duration of cows milking is modelled. The microprocessor unit is designed using a single-chip microcontroller. It ensures the algorithm set by the central computer and implements a step of changing the pulsation frequency of 0.1 Hz, the ratio between the cycles of 0.25%, the phase shift step of 0.1 s.

РЕЗЮМЕ

Розглянуто концепцію функціональної керованості молоковіддачею, яка уможливлює прогнозування інтенсивності молоковіддачі в режимі он-лайн роботи доїльного апарата. Наведено архітектуру функціональної керованості інтенсивністю молоковіддачі. Описані вхідні і вихідні параметри структурно-функціональної схеми адаптивного керування інтенсивністю молоковіддачі. Розроблена аналітична модель інтенсивності молоковіддачі на основі розподілу Пірсона. Проведено моделювання інтенсивності молоковіддачі для різної продуктивності і тривалості доїння корови. Наведена функціональна реалізація адаптивної доїльної системи та її загальний вигляд. Мікропроцесорний блок з використанням однокристального мікроконтролера забезпечує алгоритм роботи, який задається центральним комп'ютером та реалізовує крок зміни частоти пульсації 0.1 Гц, *співвідношення між тактами* 0.25 %, *крок зсуву фаз* 0.1 с.

INTRODUCTION

The adaptive system of cow machine milking implements the "machine-animal" biotechnical system, provides realization of functional controllability of milk ejection intensity of animal. The effectiveness of functional controllability of milk production depends on the parameters that ensure the quality and efficiency of technological functions.

The adaptive milking machine must ensure the adaptation of the parameters of the technical system to the physiology of milk ejection of the cow. The implementation of such a system is possible through self-regulation of technological characteristics of machine milking, which allows predicting the intensity of milk ejection of cows (*Dmytriv V.T. et al, 2020*). However, predicting the intensity of milk ejection during milking is an unrealized task. Accordingly, the prediction of technological parameters for the next cycle of the technical milking system without the parameter of milk ejection of the system is incorrect.

The relevance of predicting the intensity of milk ejection in the online mode of the milking machine is confirmed by a significant number of studies, which can be roughly attributed to the partial solution of the problem. Thus, individual variations of daily milking and duration of single milking depending on the duration of the milking interval are quantified, models of random factors are used to describe the impact of individual milking interval on daily milk yield and duration of single milking (*Andre G. et al, 2010*). It is established that the value of vacuum should be variable in the process of milk ejection.

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[1] *Dmytriv I.V., Assoc. prof., Ph.D.Eng.*

Adjusting the vacuum during milking the cow increases the intensity of milk production and reduces the duration of a single milking (*Reinemann D. J. et al, 2021*).

A number of researchers believe that the basis is the cow's lactation curve, which characterizes the productivity of the animal and the influence of key factors on the milk production process. They investigate lactation curves, which represent the theoretical dynamics of milk yield using an iterative procedure that corresponds to Wood's model based on daily milk data, and the influence of various factors on its characteristics (*Adriaens I. et al, 2020; Ben Abdelkrim A. et al, 2021*). Such modelling of the cow's lactation curve has a limitation, which is that to comply with the typical lactation curve the short-term disturbances are ignored, i.e. the significant deviations of the quantitative parameters of milk from the lactation curve are not taken into account (*Adriaens I. et al, 2018*). But the characteristics of the disturbances are important for the analysis of milk productivity of cows and, accordingly, decisions on the functional controllability of the process (*Elgersma G.G. et all, 2018*). Taking into consideration the disturbances (the amount of milk is outside the lactation curve) that occur during lactation, models of lactation with perturbation are developed that capture contrasting characteristics, from a sharp and short decline in milk yield to a long and slow decline in milk yield (*Sadoul B. et al, 2015; Nguyen Ba H. et al, 2020*; *Revilla M. et al, 2019*)

A detailed analysis of the impact of technical and technological parameters on the functional controllability of milk ejection is given in the review article: setting up of milking machines, the condition of teats and the efficiency of cows milking (*Odorcić M. et al, 2019*).

One of the parameters of the functional controllability of milk production is the fluctuations of the vacuum in the suction phase (sucking stroke). It is revealed that the amplitude of fluctuations is higher with increasing of the milk ejection; this characteristic is important for the adaptation of the system in the suction phase (*Ströbel U. et al, 2016*). It is important to determine the characteristics of the interaction amongst the compression of the teat cup liner, the milking vacuum and the duration of the pulsation b-phase. It is revealed that the increase in excessive pressure increases the hyperkeratosis, especially in highly productive cows, which are milked three or more times a day. Increasing the vacuum causes more teats congestion than increasing the b-phase in the most common range. And changes in average milk ejection are much smaller than changes in peak milk ejection, so an increase in peak milk ejection did not reflect a corresponding reduction in milking time (*Reinemann D.J. et al, 2010; Bade R. D*. *et al, 2009*). The combination of high levels of vacuum and the open phase of teat cup liner increases the potential for congestion at the tips of the teats (*Penry J.F. et al., 2017*). Conversely, a lower vacuum reduces the intensity of milk ejection and increases the duration of a single machine milking, but there is less effect on the teat tissue (*Besier J. and Bruckmaier R.M, 2016*). The determining factor in the intensity of milk ejection is the vacuum in the sucking phase and its different levels during machine milking (*Besier J. and Bruckmaier R.M., 2016*). The vacuum at the end of the sucking phase affects the teat tissue at the beginning and end of milking. Therefore, the availability of the mathematical model of vacuum change makes it possible to conduct simulation tests of the controllability of the machine milking process (*Golisz E. et al, 2021*).

Among these studies, investigations of vacuum changes in the milking system have a special place (*Achkevych O. et al, 2020; Medvedskyi O., 2018*), and this is the main technical parameter of the milking machine. The vacuum loss and fluctuations in the milking machine reduce the efficiency of milking and are factors that affect the health of the udder of cows. They cause reverse flows of milk-air mixture, which can reach significant velocities in the certain designs of the milking machine (*Dmytriv V., 2020*).

The pulsator is a component of the adaptive milking machine system. The pulsation system of the milking machine affects the intensity of milk production, the duration of milking, the condition of the teats and the milk amount. Therefore, the study of the influence of the pulsation coefficient on daily milk yield, milk production intensity and duration of milking are directly related to the functional controllability of milk ejection (*Dmytriv V.T. et al, 2019*). The milking process in the milking parlor with the MultiLactor milking system has been studied, the peculiarity is that each quarter of the udder is milked separately. It is established that the pulsation coefficient of the milking process is a parameter of adaptation and allows functional controllability of milk ejection (*Kaskous S., 2018*).

Both the irregular pulses of vacuum during milking in combination with the frequency of pulsations and high value of vacuum at the end of the teat increase the likelihood of infection of the teats and reduce the flow rate of milk (*Besier J. et al, 2016*). A higher vacuum at the end of the teat reduces the peak intensity of milk production; on teats the milking cups are moved more intensively that can lead to injuries of tips of teats, cause an infection of an udder, and also lead to accumulation of liquid and obstruction of channels on tips of teats (*Penry J.F. et al, 2017*; *Wieland M. et al, 2017*).

A number of researchers analyzed the factors that have influence on the vacuum in the milking machine, they also simulated the processes under the condition of changing the vacuum pulsations and changes in the intensity of milk production (*Enokidani M. et al, 2016*; *Enokidani M. et al, 2016*). Other scientists have studied the effect of vacuum on the efficiency of machine milking and the condition of teats depending on milk ejection (*Besier J., Bruckmaier R.M., 2016*; *Stauffer C., et al, 2020*; *Parilová M. et al, 2010*).

The analysis of researches shows that functional controllability of milk ejection is realized by parameters of vacuum, pulsation frequency and off-duty factor of impulses as a function of intensity of the milk ejection of a cow. These parameters must be set by the system in advance of the next pulse cycle. Therefore, the prediction of the next interval of milk intensity is possible at the presence of a model of the curve of milk intensity ejection for a concrete cow. Analysis of research has shown the absence of such models.

The purpose of this study is to develop models of the intensity of milk ejection of cows and a system of functional controllability of milk ejection, which will enable the implementation of an adaptive milking system and is relevant for improving the efficiency of milking systems.

MATERIALS AND METHODS

The concept of functional controllability of milk ejection

To date, the intensity of milk ejection has been characterized as a parameter of the biological state of the animal and the functional influence of the parameters (vacuum gage pressure, pulsation frequency, ratio of cycles) of the technical system. The influence of these factors on the biological intensity of milk ejection (velocity and amount of milk) was evaluated as a probabilistic process.

To implement the adaptation of the machine milking system it is necessary to consider the functional controllability of the intensity of milk ejection by the technical system. At the reflex level, technical parameters act as stimuli that affect sensation and physical action, as well as technological parameters of the process (fig. 1).

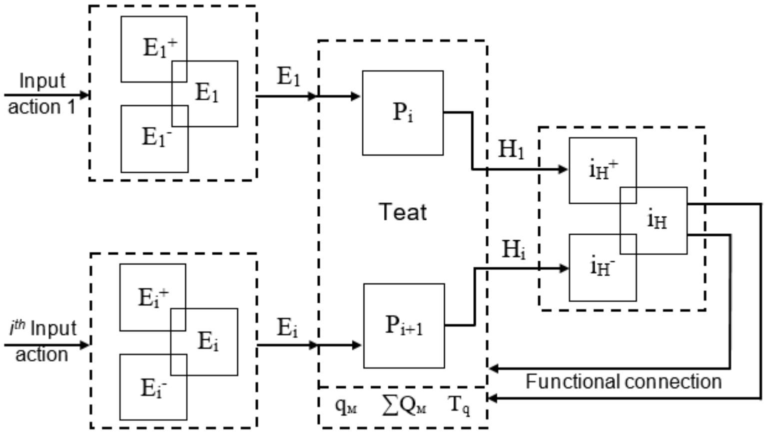


Fig. 1 - Architecture of the functional controllability of milk election intensity

$E_1^+, E_1^-, E_i^+, E_i^-$ – *limits of input parameters of the technical system;* E_1, E_i, – *optimal values of input parameters of the technical system;* P_i, P_{i+1}, – *receptors that are affected by input parameters;* H_1, H_i, – *quantitative parameter of feedback self-descriptiveness;* i_H^+, i_H^- – *limits of the effective self-descriptiveness;* i_H –*information of functional connection on technological parameters of milk ejection*

Mathematically, the characteristic of the functional controllability of the intensity of milk ejection is formed as the dependence of the intensity of milk ejection on the technological and technical parameters of the system:

$$q_m = f\left(P, \tau, \tau(t), K_{\tau_i}\right) \tag{1}$$

where P – the vacuum gage pressure; τ – the pulsation frequency;

$\tau(t)$ – the off-duty factor of impulses (ratio of cycles);

$K_{\tau i}$ – the coefficient of the constant of time of vacuum gage pressure increase and decrease in the inter wall space of the milking or teat cup.

The coefficient $K_{\tau cc}$ of the constant of time of vacuum increase (transition from the compression stroke to the suction stroke):

$$K_{\tau_{cc}} = f\left(P, \tau, \tau(t), F_H\right) \tag{2}$$

and the coefficient $K_{\tau_{CT}}$ of the constant of time of atmospheric pressure increase (transition from the suction stroke to the compression stroke):

$$K_{\tau_{CT}} = f\left(P,\tau,\tau(t),F_H\right) \tag{3}$$

where F_H – the force of mounting tension of the teat cup liner in the teat cup shell.

Acceleration of closing and opening of the teat cup liner:

$$a_\omega = f\left(P(t),F_H\right) \tag{4}$$

where $P(t)$ – function of the pressure change over time (for 1 cycle of the pulsator work).

According to the above factors of the architecture of functional control, the structure of the technological system will be developed, which will allow intensity control, as a functional adaptation to the physiology of the process of milk ejection (Fig. 2). Functional controllability of milk ejection by the technical system is realized by three groups of parameters: input (technological) parameters, are realized by the technical system and control the work of the pulsator and the control chamber of the milking or teat cup; output (information) parameters of functioning of the "teat – milking teat cup" system; information parameters, which are implemented by algorithmic-hardware methods through the microcontroller and displayed on the information board of the milking machine operator, on the central computer of the dispatcher and in the database.

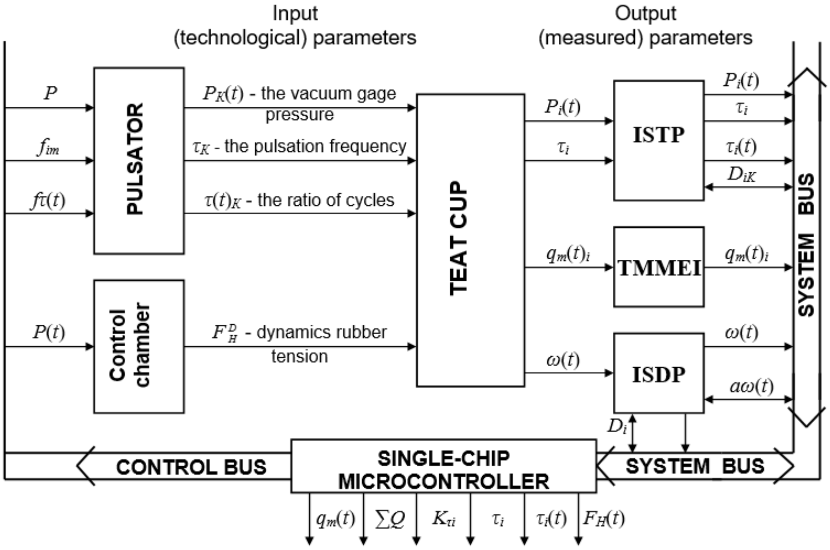


Fig. 2 - Structural and functional scheme of adaptive control of milk ejection intensity

ISTP – *information system for technological parameters determining;* ISDP – *information system for dynamic parameters determining;* TMMEI – *thermoanemometric measurer of milk ejection intensity;* τ_i – *pulsation frequency in the i-th moment of time;* $P_i(t)$ – *pressure in the i-th moment of time*; D_{iK} – *address data, form the number of the cow*; $q_m(t)_i$ – *milk ejection intensity in the i-th moment of time;* $\omega(t)$ – *speed of closing and opening of teat cup liner;* $a_\omega(t)$ – *acceleration of closing and opening of teat cup liner*.

The model of milk ejection, as an element of functional controllability of milk election intensity, is described mathematically and it is possible to predict it on the basis of a perspective assessment of this parameter at the $i + 1$ time of milking.

Mathematical model of milk ejection intensity

The intensity of the milk ejection is denoted as q by the variable of y; the duration of milk ejection will be denoted as T, which corresponds to the duration of milking. Accept $x \to t$ condition as current value of time. Let the milk ejection curve have the form shown in Fig. 3.

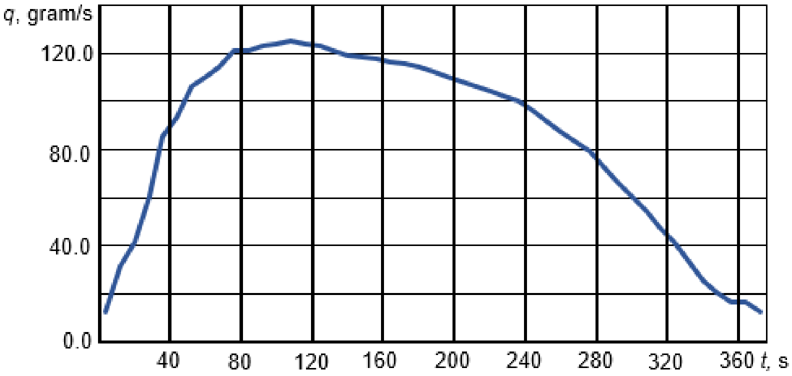


Fig. 3 – The curve of milk ejection

Considering the fact that x is a time parameter and Δx is a quantization period corresponding to the condition of $\Delta x = \mathrm{const}$, the interval of integration of milk ejection intensity will be as follows:

$$t = k_{\mathrm{i}} \cdot \Delta x \tag{5}$$

where k_i – the number of measurements for time *t*, which can be defined as $k_i = t/\Delta x$.

Then

$$n_i = y_i = \int_{x_i - \frac{t}{2}}^{x_i + \frac{t}{2}} y(x) \cdot dx = y(x_i, b_0, b_1, b_2) \tag{6}$$

To describe the milk ejection curve, Pearson distribution is used (*Korn G. et al, 1977*), which is subject to the differential equation:

$$\frac{dy}{y} = \frac{x - a}{b_0 \cdot x^2 + b_1 \cdot x + b_2} \cdot dx \tag{7}$$

The coefficients of b_0, b_1, b_2 are determined and the equation (7) is rewritten as follows:

$$x^n \cdot \left(b_0 \cdot x^2 + b_1 \cdot x + b_2\right) \cdot dy = -y \cdot x^n \cdot (x - a) \cdot dx\,. \tag{8}$$

After integrating the left side of (8) equation, the following is obtained:

$$\begin{aligned} &\left[x^n \cdot \left(b_0 \cdot x^2 + b_1 \cdot x + b_2\right) \cdot y\right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \left((n+2) \cdot b_0 \cdot x^{n+1} + (n+1) \cdot b_1 \cdot x^n + \right. \\ &\left. + n \cdot b_2 \cdot x^{n-1}\right) \cdot y \cdot dx = -\int_{-\infty}^{\infty} x^{n+1} \cdot y \cdot dx + a \int_{-\infty}^{\infty} x^n \cdot y \cdot dx \end{aligned} \tag{9}$$

Let the expression in square brackets of equation (9) be zero at the ends of the distribution curve, which characterizes the milk ejection curve, which corresponds to reality.

$$\lim_{y \to \pm\infty} \; x^{n+2} \cdot y \to 0 \tag{10}$$

The milk ejection curve is probabilistic, respectively, the moment of distribution relative to zero of the parameter of milk ejection intensity is written as follows:

$$\mu_i = \int_0^{\infty} x^j \cdot y(x) \cdot dx \tag{11}$$

where *j* = 0, 1, 2, ..., *n*.

The moment of μ_i distribution relative to zero is a numerical characteristic of a random variable $y(x)$.

Taking into account the dependence (11) and condition (10) the equation (9) will take the form:

$$(1 - n + 2 \cdot b_0) \cdot \mu_{n+1} - ((n+1) \cdot b_1 + a) \cdot \mu_n - n \cdot b_2 \cdot \mu_{n-1} = 0 \tag{12}$$

Expression (11) is analyzed for a random distribution that characterizes the cow's milk ejection graph as the probability distribution of a random variable. For *j* = 0 from dependence (11) results:

$$\mu_0 = \int_0^{\infty} y(x) \cdot dx = \sum_{i=0}^{n} y_i(x) \Big/ \sum_{i=0}^{n} y_i(x) = 1 \tag{13}$$

Mathematical expectation is the average value that characterizes the center of distribution:

$$\mu_1 = \int_0^{\infty} x \cdot y(x) \cdot dx = \frac{\sum_{i=0}^{n} x_i \cdot y_i(x_i)}{\sum_{i=0}^{n} y_i(x_i)} \tag{14}$$

Then the *j*-th moment relative to the point *a*, where the point *a* is the value of x_i, which corresponds to y_{imax} is defined as:

$$\mu_{aj} = \int_0^{\infty} (x - a)^j \cdot y(x) \cdot dx = \frac{\sum_{i=0}^{n} \left((x_i - a)^j \cdot y_i(x_i)\right)}{\sum_{i=0}^{n} y_i(x_i)} \tag{15}$$

The central moment of the j order of the $y(x)$ distribution is the following equality:

$$\mu_j = \frac{1}{\sum_{i=0}^{n} y_i(x_i)} \cdot \sum_{i=0}^{n} y_i(x) \cdot \left(x_i - \bar{x}\right)^j \qquad (16)$$

There is a relationship between the central and initial moments; taking into account the equations of (13), (14), (15) and (16), it can be written:

$$\mu_j = \sum_{j=0}^{n} (-1)^j \cdot C_n^j \cdot \mu_{aj} \cdot \left(\bar{x} - a\right)^j \qquad (17)$$

Taking of j = 0, 1, 2, 3, 4 from (17) dependence, the following equations are obtained:

$$\begin{aligned} &\mu_0 = 1; \mu_1 = 0; \mu_2 = \mu_{a2} - \left(\bar{x} - a\right)^2 \\ &\mu_3 = \mu_{a3} - 3 \cdot \mu_{a2} \cdot \left(\bar{x} - a\right) + 2 \cdot \left(\bar{x} - a\right)^3 \\ &\mu_4 = \mu_{a4} - 4 \cdot \mu_{a3} \cdot \left(\bar{x} - a\right) + 6 \cdot \mu_{a2} \cdot \left(\bar{x} - a\right)^2 - 3 \cdot \left(\bar{x} - a\right)^4 \end{aligned} \quad . \qquad (18)$$

The (7) parabola equation coefficients are determined from the (12) dependence by substituting of n = 0, 1, 2, 3 and taking into account that μ_0 = 1, μ_1 = 0 a system of equations is obtained:

$$\left.\begin{aligned} &n = 0: \quad b_1 + a = 0; \\ &n = 1: \quad 3 \cdot b_0 \cdot \mu_2 + b_2 - \mu_2 = 0; \\ &n = 2: \quad 4 \cdot b_0 \cdot \mu_3 + 3 \cdot b_1 \cdot \mu_2 + a \cdot \mu_2 - \mu_3 = 0; \\ &n = 3: \quad 5 \cdot b_0 \cdot \mu_4 + 4 \cdot b_1 \cdot \mu_3 + 3 \cdot b_2 \cdot \mu_2 + a \cdot \mu_3 - \mu_4 = 0 \end{aligned}\right\} \qquad (19)$$

From the system of equations (19), the coefficients of b_0, b_1, b_2 are calculated:

$$\begin{aligned} b_0 &= \frac{2 \cdot \mu_4 \cdot \mu_2 - 3 \cdot \mu_3^2 - 6 \cdot \mu_2^3}{2 \cdot \left(5 \cdot \mu_4 \cdot \mu_2 - 6 \cdot \mu_3^2 - 9 \cdot \mu_2^3\right)} \\ b_1 &= \frac{\mu_3 \cdot \mu_4 + 3 \cdot \mu_3 \cdot \mu_2^2}{2 \cdot \left(5 \cdot \mu_4 \cdot \mu_2 - 6 \cdot \mu_3^2 - 9 \cdot \mu_2^3\right)} \\ b_2 &= \frac{4 \cdot \mu_4 \cdot \mu_2^2 - 3 \cdot \mu_2 \cdot \mu_3^2}{2 \cdot \left(5 \cdot \mu_4 \cdot \mu_2 - 6 \cdot \mu_3^2 - 9 \cdot \mu_2^3\right)} \end{aligned} \qquad (20)$$

After determining the coefficients of (7) equation, the differential equation is integrated:

$$\int \frac{dy}{y} = -\int \frac{x - a}{b_0 \cdot x^2 + b_1 \cdot x + b_2} \cdot dx \qquad (21)$$

To do this, the (21) equation is decomposed in the denominator of the differential into a multiplier as follows:

$$b_0 \cdot x^2 + b_1 \cdot x + b_2 = b_0 \cdot \left(x^2 + \frac{b_1}{b_0} \cdot x + \frac{b_2}{b_0}\right) = b_0 \cdot \left(x + k_1\right) \cdot \left(x - k_2\right) \qquad (22)$$

where k_1, k_2 – the roots of the equation and

$$k_{1,2} = \frac{-\frac{b_1}{b_0} \pm \sqrt{\left(\frac{b_1}{b_0}\right)^2 - 4 \cdot \frac{b_2}{b_0}}}{2}$$

Assume that the roots of k_1, k_2 are natural numbers, and one of them can have a negative value. Then, the subintegral expression of (21) dependence is written in the following form:

$$-\frac{x - a}{b_0 \cdot x^2 + b_1 \cdot x + b_2} = \frac{1}{b_0} \cdot \left(\frac{A_1}{x + k_1} + \frac{A_2}{x - k_2}\right) \qquad (23)$$

where $A_1 = -\frac{a + k_1}{k_1 + k_2}$; $A_2 = \frac{a - k_2}{k_1 + k_2}$.

After entering the notation of $q_1 = A_1/b_0$; $q_2 = A_2/b_0$ the (21) dependence will take the following form:

$$\int \frac{dy}{y} = \int \frac{q_1}{x+k_1} \cdot dx + \int \frac{q_2}{x-k_2} \cdot dx, \qquad \ln y = q_1 \cdot \ln(x+k_1) + q_2 \cdot \ln(x-k_2) + C. \tag{24}$$

Let $x = 0$; $y = y_0$, then the constant integration will be equal:

$$C = \ln \frac{y_0}{k_1^{q_1} \cdot (-k_2)^{q_2}} \tag{25}$$

Then, after minor transformations, the equation of the parabola is obtained:

$$y = y_0 \cdot \left(1 + \frac{x}{k_1}\right)^{q_1} \cdot \left(1 - \frac{x}{k_2}\right)^{q_2} \tag{26}$$

In (26) equation the y_0 is unknown. We will integrate by x:

$$\int_{-k_1}^{k_2} y \cdot dx = \sum_{i=0}^{A} y_i, \text{ accordingly } \int_{-k_1}^{k_2} y_0 \cdot \left(1 + \frac{x}{k_1}\right)^{q_1} \cdot \left(1 - \frac{x}{k_2}\right)^{q_2} \cdot dx = \sum y_i \tag{27}$$

The values in the equation are substituted:

$$z = \frac{k_1 + x}{k_1 + k_2} \Rightarrow x = z \cdot (k_1 + k_2) - k_1, \; dz = \frac{dx}{k_1 + k_2} \Rightarrow dx = dz \cdot (k_1 + k_2)$$

Under $x = -k_1 \Rightarrow z = 0$; $x = k_2 \Rightarrow z = 1$

Then, the integral of (27) will take the form:

$$\frac{y_0 \cdot (k_1 + k_2)^{q_1 + q_2 + 1}}{k_1^{q_1} \cdot k_2^{q_2}} \cdot \int_0^1 z^{q_1} \cdot (1-z)^{q_2} \cdot dz \tag{28}$$

The subintegral expression of the (28) dependence is an Euler integral that has such a solution:

$$\int_0^1 z^{q_1} \cdot (1-z)^{q_2} \cdot dz = B \cdot (q_1 + 1, q_2 + 1) \tag{29}$$

where $B \cdot (q_1 + 1, q_2 + 1)$ – beta function:

$$B \cdot (q_1 + 1, q_2 + 1) = \frac{\Gamma \cdot (q_1 + 1) \cdot \Gamma \cdot (q_2 + 1)}{\Gamma \cdot (q_1 + q_2 + 2)} \tag{30}$$

where Γ – gamma function.

Gamma functions are determined by the classical method (*Korn G. et al.1977*). Taking into consideration the dependences of (27), (28), (29) and (30), the dependence to determine the value of y_0 is obtained:

$$y_0 = \frac{\sum y_i \cdot k_1^{q_1} \cdot k_2^{q_2}}{(k_1 + k_2)^{q_1 + q_2 + 1}} \cdot \frac{\Gamma \cdot (q_1 + q_2 + 2)}{\Gamma \cdot (q_1 + 1) \cdot \Gamma \cdot (q_2 + 1)} \tag{31}$$

With the dependence of (31), the (26) expression will look like:

$$y = \frac{\sum y_i \cdot k_1^{q_1} \cdot k_2^{q_2}}{(k_1 + k_2)^{q_1 + q_2 + 1}} \cdot \left(1 + \frac{x}{k_1}\right)^{q_1} \cdot \left(1 - \frac{x}{k_2}\right)^{q_2} \cdot \frac{\Gamma \cdot (q_1 + q_2 + 2)}{\Gamma \cdot (q_1 + 1) \cdot \Gamma \cdot (q_2 + 1)} \tag{32}$$

The equation (32) characterizes the milk ejection curve of a cow at the process of milking.

RESULTS

Realization of the milk ejection intensity model

To realize the model of milk ejection intensity in the form of the (32) function, an arbitrary graphical dependence is considered in (Fig. 3). This graphical dependence of milk ejection can be interpreted as follows. The milk ejection interval is divided evenly by 5 s, the total duration of milk ejection at cow milking is 378 s, or 6.3 min. Accordingly, x_i is the middle of the x time interval. Then the intensity of milk ejection in the x time interval will correspond to x_i and will be $y(q)_i$, gram/s, respectively. In fact, $y(q)_i$ characterizes the weight quantitative coefficient of the x_i argument. Then the $\bar{x}$ average value is calculated by dependence:

$$\bar{x} = \frac{1}{\sum y(q)_i} \cdot \sum \left(y(q)_i \cdot (x_i - a) \right) + a \tag{33}$$

where a – arbitrary value of x.

Under the condition of a = 67.5 at x = 5 s is obtained $\bar{x} = 74.746$. Accordingly, the moments with respect to zero will be as follows: $\mu_0 = 1$; $\mu_1 = 0$; $\mu_2 = 3949.956$; $\mu_3 = 192861.042$; $\mu_4 = 28023477.44$.

Due to the known values of the central moments of the μ_0, μ_1, μ_2, μ_3, μ_4, orders, the coefficients of the parabola equation are calculated by the dependences of (20).

The coefficients of the equation are $b_0 = 0.46384$; $b_1 = -31.81023$; $b_2 = -1644.10432$.

The (34) equation of the parabola is solved, assuming that $y = 0$:

$$y = 0.46384 \cdot x^2 - 31.81023 \cdot x - 1644.10432 \tag{34}$$

The roots of the (34) quadratic equation are as follows: $K_1 = 102.995$; $K_2 = -34.415$. According to the calculation results, the following values were obtained: $q_1 = -5.36$; $q_2 = 3.204$; Gamma functions: $\Gamma(q_1+q_2+1) = -7.16827$; $\Gamma(q_1+1) = -0.08295$; $\Gamma(q_2+1) = 7.7975$. Then the (32) equation will look like:

$$y(q) = 0.08356 \cdot \sum y_i \cdot \frac{(34.415 + t)^{3.204}}{(102.995 + t)^{5.36}} \tag{35}$$

where $\sum y_i$ – the maximum amount of milk moved out by the milking machine for the duration of milking, gram;

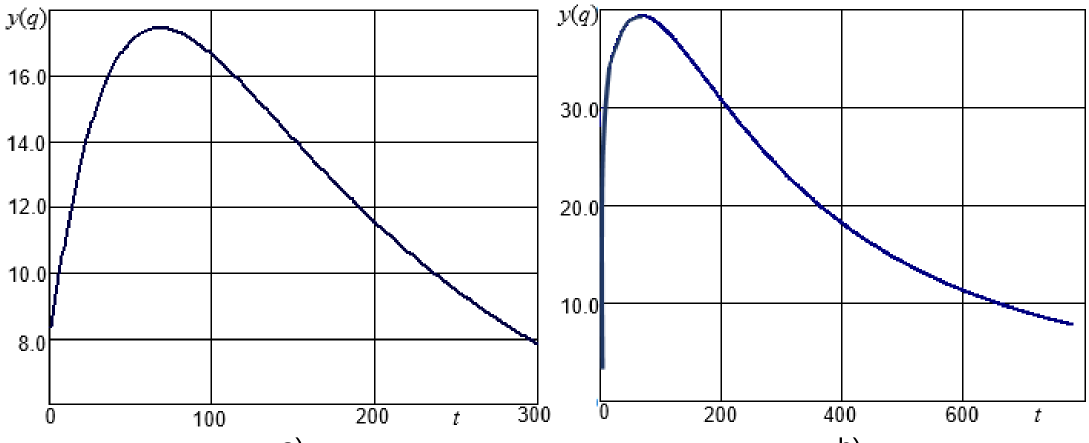
t – running time of the milking, s;

$y(q)$ – milk ejection intensity, gram/s.

For the total amount of milk for one milking of a cow $\sum y_i = 4000$ grams and duration of one-time milking by the milking machine $\sum t_i = 300$ s, according to (35) dependence the graph of milk ejection corresponds to fig. 4,a.

For the total amount of milk for one milking of $\sum y_i = 16000$ gram and milking time of $\sum t_i = 780$ [s] the parabolic equation is derived (36) and milk production graph is constructed (fig. 4,b)

$$y(q) = 2.63 \cdot 10^4 \cdot \sum y_i \cdot \frac{(26.708 + t)^{0.703}}{(361.555 + t)^{3.201}} \tag{36}$$



a) b)

Fig. 4 – Graphical realization of milk ejection intensity described by equations: a – (35); b – (36)

y(q) – intensity of milk ejection, gram/s; t – time of milking, s

The realization of functional controllability of the adaptive milking system

Functional controllability of milk ejection is realized on the basis of adaptive milking system. The intensity of milk ejection was measured with a thermoanemometric meter and compared with the calculated one. The adequacy of the parameters of the milking process was evaluated. Vacuum in the inter wall chamber of teat cup and in the under teat space (milking chamber of teat cup), the pulsations frequency and the off-duty factor of impulses were set on the basis of forecasting the intensity of milk ejection per stroke of the milking machine. These parameters create the mode of operation which is adequate to the intensity of milk ejection.

The functional scheme of the adaptive milking system is shown in Fig. 5 (*Dmytriv V.T. et al, 2020*). A general view of the hardware implementation for the study of functional controllability of the milk ejection intensity of the technical system is shown in Fig. 6.

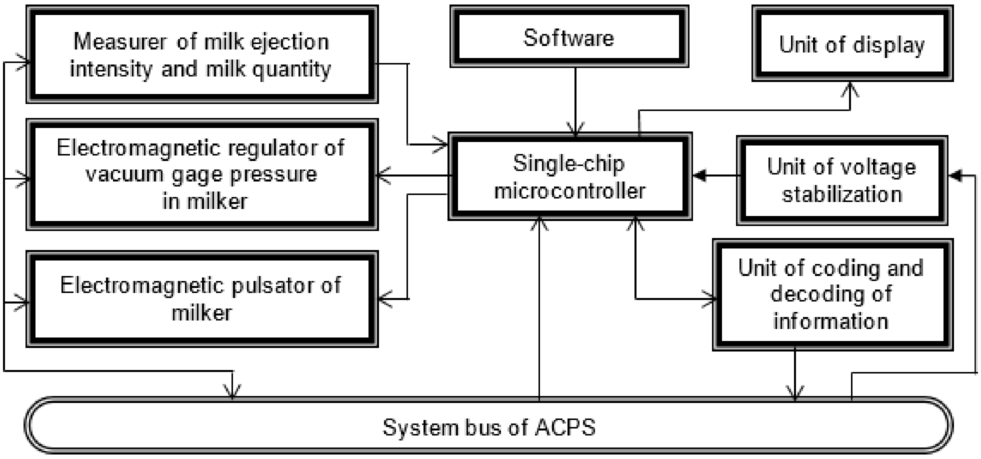


Fig. 5 – Functional implementation of adaptive milking system

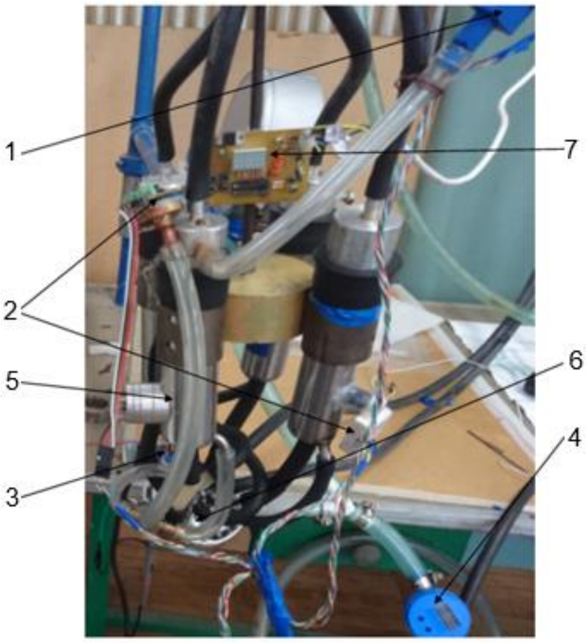


Fig. 6 – General view of an adaptive milking system for the study of the controllability by milk ejection

1 – pressure sensor in the artificial udder; 2 – pressure sensor in the inter wall chamber of the milking (teat) cup;
3 – pressure sensor in the milking chamber of the teat cup; 4 – pressure sensor in the milk hose; 5 – adaptive teat cup;
6 – claw of the milker; 7 – microprocessor control unit

The microprocessor unit is designed using the Atmel single-chip microcontroller. It ensures the algorithm set by the central computer and implements a step of changing the pulsation frequency of 0.1 Hz, the ratio between the cycles of 0.25%, the phase shift step of 0.1 s.

A digital control system was used to implement the selection of optimal modes. The functional scheme of the adaptive control unit with a search engine for self-tuning is implemented on base of the three single-chip microcontrollers (OMK). The distribution of functions is as follows: the first OMK - the functions of the mathematical model, the identification of the object of control, evaluation of the quality of control and the formation of control tasks; the second OMK - direct control of the process of machine milking, distribution of functions among other OMK; the third OMK - removal of the information on parameters of control object, optimization of parameters of adjustment of the executive elements, adjustment of parameters, data transfer, indication of the operational parameters.

CONCLUSIONS

The considered aspects of functional controllability of milk ejection of the adaptive milking system allow to state that the new generation of milking systems will be based on the new organizational and technological decisions with use of the cybernetic principles of organization of hardware-programmed functional autonomous technological elements.

Adaptation of the milking system is carried out in an automated mode, based on the parameters of the milk ejection. For each cow, an individual algorithm for the intensity of milk ejection is selected. The flexible relationship between the intensity of milk ejection of the cow and the design and technological parameters of the milking system is provided that allows an individual approach to the function of milk ejection for each cow separately.

The use of digital control systems based on single-chip microcontrollers provides the implementation of digital controllers, which significantly simplifies the hardware. The process of determining the optimal parameters for adjusting the controllers is carried out by mathematical dependence, which reduces the over-regulation of the parameters of the controllers.

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