# THEORETICAL CONSIDERATIONS ON THE OPTIMIZATION OF THE WORKING PROCESS OF VARIABLE WIDTH PLOUGHS

## CONSIDERAȚII TEORETICE PRIVIND OPTIMIZAREA PROCESULUI DE LUCRU AL PLUGURILOR CU LĂȚIME VARIABILĂ

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**Keywords:** plough, qualitative indices, multivariable regression functions

#### **ABSTRACT**

The paper presents theoretical research conducted for the optimization of the working process of variable width ploughs. Thus, were determined multivariable regression functions for the traction force of the plough with variable working function of the control parameters: working depth, working width and working speed. The use of these theoretical considerations of the optimal points sought lead to the opportunity of making assessments on the possibilities to conduct their experimental validation. Conditions for an experimental plan were formulated to highlight such optimal points and the theoretical results were validated through experiments.

#### **ABSTRACT**

Lucrarea prezintă cercetări teoretice realizate pentru optimizarea procesului de lucru al plugurilor cu lățime variabilă. Astfel, au fost determinate funcțiile de regresie multivariabilă pentru forța de tracțiune a plugului cu lățime de lucru variabilă în funcție de parametrii de control: adâncimea de lucru, lățimea de lucru și viteza de lucru. Utilizarea acestor considerații teoretice ale punctelor optime căutate duce la oportunitatea de a face evaluări cu privire la posibilitățile de efectuare a validării lor experimentale. Condițiile pentru un plan experimental au fost formulate pentru a evidenția astfel de puncte optime, iar rezultatele teoretice au fost validate prin experimente.

#### INTRODUCTION

Tillage is the primary and most energy consuming operation in farming operations. The purpose of tilling the land is to ensure favourable soil conditions by cutting and turning the soil for further seeding or transplanting (*Mouli et al., 2018*). Tillage involves the operations starting from cutting and breaking, followed by turning or moving the top layer of soil, this normally being performed in a single pass. The aim is to create a desired final state from the initial conditions of the soil by mechanical manipulation. This mechanical operation requires a significant amount of energy, not only because of the large amount of the soil mass that must be moved but also because of the selection of the tillage tool and the fixation of its depth and speed of work (*Ibrahimi et al., 2017*).

The mouldboard plough is normally the most used tillage equipment in the world. The variable width plough is a very useful farm equipment for performing the tillage work, being directly mounted on the tractor's PTO. It works on both the left and right side having a bilateral turnover mechanism that automatically reverses the plough position (*Yin et al, 2018*). The mechanical function of ploughs is to cut the soil layer and turn it to the side. Thus, it is possible to incorporate and mix fertilizers and plant residues in the soil. The use of ploughs in wet seasons will prepare the soil for good ventilation, will help retain water and create furrows resistant to erosion (*Luo et al., 2019, Zhu et al., 2016*).

Researches were conducted by numerous researchers in the field, regarding the optimum working conditions for soil processing machinery (ploughs, chisels, loosening equipment) in the last years, from 2007-2016, studying the use of ante-mouldboard in the construction of ploughs (*Biriş S.Şt. et al., 2007*), soil particles kinematics during the tillage working process using ante-mouldboard tools (*Biriş S.Şt. et al., 2008*), determining the stress distribution that appears on the lamellar surface of the mouldboards for modelling and optimization (*Bungescu S. et al., 2008*), nonlinear friction and resistance, generating sources of optimal points in the energy field of agricultural aggregates working process (*Cârdei P. et al., 2017*), or the determination of subsoilers drag force influenced by different working depths and speed (*Croitoru Şt. et al., 2016*).

Many attempts have been also made to have a better understanding of the complex nature of the tillage process using conventional or reversible ploughs (*Abbaspour-Gilandeh M. et al., 2020; Akbarnia et al., 2014; Godwin et al., 2007; Irshad Ali et al., 2015; Lortz et al., 2021*).

#### **MATERIALS AND METHODS**

The processing of experimental data is made in order to obtain multivariable regression functions of polytropic or polynomial form that allow the appreciation of the functional, energetic and qualitative indices of the plough with variable working width. During the experimental tests, independent variable elements were modified: travel speed, working depth and working width.

It was set out to determine the multivariable regression functions for the traction force of the plough with variable working width, dependent of slippage of the tractor's drive wheels, variation of working depth, variation of working width and variation of the working speed.

For their calculation, the analytical expressions of some multivariable functions will be determined, defined as a function of the type:

$$y=f(x_{i},a_{0},a_{i},a_{ii},a_{ij})$$
 (1)

expressing the dependence of the function y on the independent variables  $x_i$  and on the constants  $a_{ij}$ ,  $a_{ij}$ ,  $a_{ij}$ ,  $a_{ij}$ . Due to the complexity of solving the problem, it is necessary to go through several stages: drawing up an adequate program for organizing experiments; determining the values of the constants; testing the significance of variables, testing the adequacy of the function's form (*Constantinescu I.*, 1980).

The stages of a statistical program for organizing experiments are:

- formulating the problem and establishing the objectives of the experimental program;
- · choosing independent variables that influence the dependent variable;
- establishing the range of variation (possible technologically) for each independent variable;
- choosing, in certain intervals, some values (called levels) that the independent variables will have;
- · determining the size of the experimental error by repeating several experiments in the central point;
- execution of specific experiments in the program in a random order;
- · measurement of the dependent variable for each experiment;
- statistical analysis of data and obtaining functional relationships between independent and dependent variables;
  - interpretation of results of the statistical analysis.

The experimental research programs used to determine the function y are structured according to the following elements:

- the number  $n_*$  of experiments performed for different values of the independent variables, necessary for determining the regression coefficients;
- the number of experiments performed for identical values of the independent variables, necessary to determine the experimental error;
- · levels of independent variables;
- the content of the experiments.

The total number of experiments is:

$$n=n_*+n_0 \tag{2}$$

where  $n_0$  is the number of identical experiments required to determine the experimental errors.

The number of levels is calculated using the relation (*Constantinescu A., 2011*):

$$N \ge {}^{m_1}\sqrt{n_*} \tag{3}$$

where  $m_1$  is the total number of independent variables contained by the function.

Following is presented the calculation algorithm used to determine the regression coefficients using the least squares method.

To determine the values of the unknown regression coefficients, denoted generically with  $k_i$ , through linear mathematical regression, the functionals of the form  $T(a_i,x_j)$  were formed, as a sum of the squares of the differences between the values obtained by applying the mentioned equations and the real values measured in experiments where  $i=1\div n$ , n being the number of unknown coefficients and  $j=1\div m$ , m being the number of measured quantities.

$$T = \sum (y(x_i) - y_i)^2 \tag{4}$$

where  $y_i$  is the vector of the independent variable measured in the experimental tests, and  $y(x_i)$  is the vector of the independent variable calculated.

To determine the coefficients by mathematical regression, the least squares method was used, imposing the condition that the function T be minimal.

The minimum of the function T in relation to  $a_i$  is obtained by cancelling the partial derivatives of  $T_q$  with respect to the same coefficients, namely  $\frac{\partial T}{\partial a_i} \! = \! 0$ 

The partial derivatives of the functional T were determined according to each of them and the unique determined system was created, of n equations with n unknowns:

$$\left\{\frac{\partial T}{\partial a_i} = 0, \quad i = 1 \div n\right\} \tag{5}$$

To solve it numerically, the equations of the system were explained and the constants were eliminated to obtain the equivalent form that can be written as a matrix product:

$$ZY=X$$
 (6)

where: Z is the matrix of the system, X is the matrix of free terms and Y is the matrix of unknown coefficients  $a_i$ ,  $Y=(a_i)$ .

The determination of the vector Y formed by the unknown coefficients, was done by the numerical solving by mathematical regression of the equation (6) through the inverse matrix method, using the data strings obtained from the experiments.

$$Y = Z^{-1}X \tag{7}$$

To solve equation (6), a matrix calculation program in Mathcad was used. By replacing the coefficients in the analytical formulas, resulted the numerical forms of these functions.

Testing the significance of the coefficients is done using the Fischer Test which is a parametric test that verifies the equality of the dispersions of two normally distributed independent variables. To test the significance of the coefficients using the Fisher test, the sum of the squares of the experimental errors is calculated:

$$S_{e} = \sum_{i=n_{*}+1}^{n} \left( y_{i} + \sum_{i=n_{*}+1}^{n} \frac{y_{i}}{n_{0}} \right)^{2}$$
 (8)

The sums for the coefficients are:

$$S_{o} = n \cdot b_{o}^{2}$$

$$S_{j} = a_{j}^{2} \sum_{i=1}^{n} X_{ij}^{2}, j = 1, 2, 3, ... m_{1}$$
(9)

The ratios are calculated:

$$F_{o} = \frac{S_{o}(n_{o}-1)}{S_{e}}$$
 (10)  
$$F_{j} = \frac{S_{j}(n_{o}-1)}{S_{e}}$$
  $j = 1,2,3,...,m_{1}$ 

If  $F_0 \ge F$  (1- $\alpha$ , 1,  $n_0$  - 1) and  $F_j \ge F$  (1- $\alpha$ , 1,  $n_0$  - 1) coefficient  $a_0$  respectively coefficients  $a_j$  are significant. If this condition is not met, for one or more coefficients, the respective coefficients are equal to zero. Critical values  $F(P = 1 - \alpha, k_1 = 1, k_2 = n_0 - 1)$  are given (*Reich R.*, 1978) for a level of significance  $\alpha = 0.95$ .

To test the adequacy of the form of the function with the Fisher test, the ratio is calculated:

$$F = \frac{(S - S_e)(n_0 - 1)}{S_e(n - n_0 - m_1)} < F(1 - \alpha, n_* - m_1, n_0 - 1)$$
(11)

where  $n^*$  represents the number of different experiments, and  $m_1$  is the number of function coefficients (without the coefficient  $a_0$ ). If the condition is met then the form of the function is adequate (*Popescu & Badescu*, 2000).

#### **RESULTS**

### Expression of traction force by multivariate polynomial functions

In order to determine the coefficients of the multivariable functions, the independent variables that influence the dependent variable and their variation interval were chosen:

- Working depth: a = 0.1 0.3 m;
- Working speed: v = 0.9 − 1.9 m/s;
- Working width: B=0.8 1.2 m, which corresponds to a working width of the plough body, b = 0.2...0.4 m.

The experimental test program for determining the multivariable functions for the traction force is presented in table 1.

Experimental results for determining the traction force

Table 1

Table 2

Sample	Slippage	Working	Working depth ai	Speed v <sub>i</sub>	Average traction
no.	coefficient $\delta_i$ [%]	width B [m]	[m]	[m/s]	force Ft [N]
1	13.80	0.80	0.10	0.938889	17150
2	15.20	0.83	0.12	1.569444	17560
3	16.70	0.83	0.13	1.886111	17590
4	14.40	0.83	0.20	0.908333	20960
5	16.50	0.83	0.22	1.347222	21170
6	17.70	0.83	0.23	1.816667	22100
7	15.80	0.82	0.30	0.886111	23530
8	17.20	0.83	0.33	1.25	23850
9	18.80	0.83	0.32	1.469444	24130
10	14.60	1.01	0.10	0.891667	18240
11	15.80	1.04	0.12	1.472222	19150
12	17.90	1.03	0.13	1.861111	19970
13	15.20	1.04	0.20	0.877778	21760
14	17.70	1.04	0.21	1.294444	22060
15	18.30	1.04	0.23	1.733333	22950
16	16.20	1.04	0.30	0.805556	23560
17	20.40	1.04	0.31	1.188889	24320
18	23.70	1.04	0.32	1.597222	24480
19	15.70	1.20	0.10	0.869444	20980
20	18.00	1.23	0.12	1.447222	21160
21	19.10	1.23	0.11	1.741667	21440
22	16.80	1.23	0.20	0.875	24280
23	18.90	1.23	0.21	1.269444	24320
24	19.80	1.22	0.20	1.694444	24720
25	17.40	1.23	0.30	0.797222	26240
26	21.20	1.23	0.31	1.138889	26840
27	24.20	1.23	0.30	1.744444	28130

The matrix of the correlations of the process parameters measured during the experiments is given in Table 2. It was constructed by calculating the correlation coefficient for each pair of data strings representing the columns of Table 1.

Matrix of correlation coefficients of the parameters measured during the experiments

	F [N]	a [m]	b [m]	v [m/s]	δ <sub>v</sub> [%]
F [N]	1.000	0.839	0.486	-0.073	-0.717
a [m]	0.839	1.000	0.000	-0.223	-0.504
b [m]	0.486	0.000	1.000	-0.061	-0.449
v [m/s]	-0.073	-0.223	-0.061	1.000	-0.487
δ <sub>v</sub> [%]	-0.717	-0.504	-0.449	-0.487	1.000

The values of the elements of the correlation matrix led to very important conclusions for the analysis of experimental data.

- The traction force is strongly correlated (directly) with the working depth and is significantly correlated (directly) with the working width, and strongly correlated (inversely) with the skidding;
- The traction force is not significantly correlated with the working speed, at least in the experimental interval.

The first form of the traction force function is the Goreachkin function (12), in which the values of coefficients f, k and  $\epsilon$ , which are considered to be constant, will be numerically determined. Also, G, the weight of the plough-tractor unit used in the experiments, is constant.

$$F_t = fG + kaB + \epsilon aBv^2 \tag{12}$$

where:  $F_t$  the draft force, f is a coefficient of friction between metal and soil, k is a coefficient that characterizes the specific deformation resistance of the soil, a is the working depth, B=nb is the plough working width where n is the number of mouldboards,  $\epsilon$  is a coefficient that depends on the surface of the active shape of the mouldboard and to the soil properties, and v is the working speed.

To determine the values of the coefficients f, k and  $\epsilon$  by linear mathematical regression, the functional  $T(f, k, \epsilon, G, a_i, B_i, v_i)$  was formed as a sum of the squares of the differences between the values obtained by applying equation (1) and the real values measured during experiments  $a_i$ ,  $B_i$ ,  $v_i$  from Table 1.

$$T = \sum (F - F_i)^2 = \sum (fG + ka_iB_i + \epsilon a_iB_iv_i^2 - F_i)^2 \rightarrow min$$
(13)

To determine coefficients f, k and  $\epsilon$ , by mathematical regression, the condition was imposed that T expressed by equation (13) be minimal.

The minimum of the function T in relation to f, k and  $\epsilon$  is obtained by cancelling the partial derivatives of T in relation to the same coefficients, namely  $\frac{\partial T}{\partial f} = 0$ ,  $\frac{\partial T}{\partial k} = 0$  and  $\frac{\partial T}{\partial \epsilon} = 0$ .

The partial derivatives of the functional T were determined according to each of them and the unique determined system was created, of 3 equations with 3 unknowns:

$$\begin{cases} \frac{\partial T}{\partial f} = 2\sum (fG + ka_iB_i + \epsilon a_iB_i{v_i}^2 - F_i)G = 0\\ \frac{\partial T}{\partial k} = 2\sum (fG + ka_iB_i + \epsilon a_iB_i{v_i}^2 - F_i)a_iB_i = 0\\ \frac{\partial T}{\partial \epsilon} = 2\sum (fG + ka_iB_i + \epsilon a_iB_i{v_i}^2 - F_i)a_iB_i{v_i}^2 = 0 \end{cases}$$

$$(14)$$

For the numerical solving of the system, the constants were eliminated and the equivalent form was obtained which can be written as a matrix product:

$$\begin{cases} f \sum G^{2} + k \sum Ga_{i}B_{i} + \epsilon \sum Ga_{i}B_{i}v_{i}^{2} = \sum GF_{i} \\ f \sum Ga_{i}B_{i} + k \sum a_{i}^{2}B_{i}^{2} + \epsilon \sum a_{i}^{2}B_{i}^{2}v_{i}^{2} = \sum F_{i}a_{i}B_{i} & \leftrightarrow ZY = X \\ f \sum Ga_{i}B_{i}v_{i}^{2} + k \sum a_{i}^{2}B_{i}^{2}v_{i}^{2} + \epsilon \sum a_{i}^{2}B_{i}^{2}v_{i}^{4} = \sum F_{i}a_{i}B_{i}v_{i}^{2} \end{cases}$$
(15)

where:

$$Z = \begin{pmatrix} \sum G^{2} & \sum Ga_{i}B_{i} & \sum Ga_{i}B_{i}v_{i}^{2} \\ \sum Ga_{i}B_{i} & \sum a_{i}^{2}B_{i}^{2} & \sum a_{i}^{2}B_{i}^{2}v_{i}^{2} \\ \sum Ga_{i}B_{i}v_{i}^{2} & \sum a_{i}^{2}B_{i}^{2}v_{i}^{2} & \sum a_{i}^{2}B_{i}^{2}v_{i}^{4} \end{pmatrix}$$

$$X = \begin{pmatrix} \sum GF_{i} \\ \sum F_{i}a_{i}B_{i} \\ \sum F_{i}a_{i}B_{i}v_{i}^{2} \end{pmatrix}$$

$$Y = \begin{pmatrix} f \\ k \end{pmatrix}$$
(16)

The determination of the vector Y formed by the unknown coefficients  $(f, k, \epsilon)$  that need to be calculated, was done by numerical solution through mathematical regression of equation (16), obtained from the matrix equation (15) by the inverse matrix method, using the data strings obtained from experiments.

$$Y = Z^{-1}X \tag{17}$$

Table 1 shows the values of the independent and dependent variables used in the mathematical regression operation performed using a calculation program in Mathcad.

The weight of the plough-tractor unit G was constant, 49830 N, corresponding to a mass of the tractor of 4480 kg and of the plough of 600 kg.

Using a calculation program in Mathcad, from equation (12) and the experimental data from table 1, coefficients  $(f, k, \epsilon)$  were determined, resulting in the mathematical model (13) of the function.

$$Ft = 0.317G + 27850aB + 1188aBv^2$$
 (18)

where Ft represents the traction force.

The deviations of the calculated values from the experimental ones for the traction force are calculated with the relation:

$$A = \frac{|F_{ti} - F_{ci}|}{F_{ti}} \cdot 100, \quad [\%]$$
 (19)

where Fci is the vector of traction forces calculated using relation 13

Table 3 shows the values of the traction force measured during the experiments and the values of the traction force calculated using relation (19) and the deviation.

Table 3

Deviations of the calculated values from the experimental ones for the traction force

Sample	Average measured	Average measured	
no.	traction force, Fi [N]	traction force, Fci [N]	A [%]
1	17150	18108.655	5.59
2	17560	18862.062	7.415
3	17590	19257.692	9.481
4	20960	20582.413	1.801
5	21170	21275.614	0.499
6	22100	21861.364	1.08
7	23530	22876.921	2.776
8	23850	23932.717	0.347
9	24130	23874.419	1.059
10	18240	18705.061	2.55
11	19150	19593.696	2.317
12	19970	20076.665	0.534
13	21760	21779.665	0.09
14	22060	22313.546	1.149
15	22950	23311.649	1.576
16	23560	24725.883	4.949
17	24320	25316.234	4.096
18	24480	26072.942	6.507
19	20980	19246.519	8.263
20	21160	20274.505	4.185
21	21440	20052.263	6.473
22	24280	22871.204	5.802
23	24320	23484.382	3.436
24	24720	23423.814	5.243
25	26240	26351.243	0.424
26	26840	27002.596	0.606
27	28130	27406.275	2.573

The maximum deviation thus determined between the experimental data and the calculated data was 9.481%, resulting in a good accuracy of the proposed model. The correlation coefficient calculated using formula (12) in which the string X was replaced with Fi and the string Y with Fci was 0.946, demonstrating a very strong correlation between the two strings.

If conditions are imposed that include some of the variables of the traction force, conditions from which a variable is removed, then, in some cases, functions that have extreme values depending on some of the remaining variables result. This was achieved using as an objective function the traction force (Goriachkin variant) and the condition to reach a given productivity.

Thus, we have the traction force resulting from the interpolation of the experimental data using the least squares method, as well as function (18) and the additional condition:

$$W = B \cdot v = const \tag{20}$$

By eliminating the working speed between relations (18) and (20), a traction force function is obtained depending on the working width of the plough with variable width and on the working depth.

$$Ft = 0.317G + 27850aB + \frac{1188aW^2}{B}$$
 (21)

Figure 1 shows the variation of the traction force depending on the working width, for three fixed productivities and the working depth a = 0.3 m in the case of the plough with variable working width.

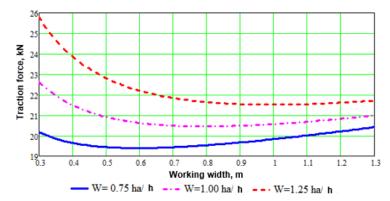


Fig. 1 - Variation of traction force with working width, for three productivities

It is observed that the minimum points of the traction force move with the increase of the programmed productivity, in the sense of increasing the optimal working width and, obviously, the optimal force.

The values of the optimal widths of the traction force and of the working width of the plough are calculated with relations:

$$F_{\text{opt}} = fG + 2aW\sqrt{k\varepsilon}$$
 (22)

$$B_{\rm opt} = W \sqrt{\frac{\varepsilon}{k}} \tag{23}$$

For the three productivities used (fig. 1), the optimal points are obtained with the coordinates given in table 4.

Table 4

Coordinates of the optimal points of the curves in Fig. 1

Productivity	Optimal width	Optimal traction width
W [ha/h]	B <sub>opt</sub> [m]	F <sub>opt</sub> [N]
0.75	0.596	19391
1.00	0.794	20453
1.25	0.993	21516

Figure 2 shows in an orthogonal coordinate system the variation of the traction force as a function of the working width for the required working productivity.

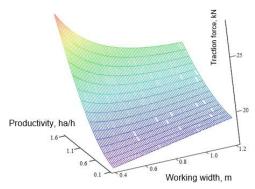


Fig. 2 - Variation of the traction force depending on the working width for the required productivity

As can be seen, the dependence of the traction force on the required productivity is not linear. It can also be seen see that there is a linear dependence between the working width and the traction force.

Further, the traction force is presented as a second-degree polynomial function, dependent on working depth and width, working speed and slippage:

$$F_t = c_0 + c_1 a + c_2 B + c_3 v + c_4 \delta + c_{12} a B + c_{13} a v + c_{14} a \delta + c_{23} B v + c_{24} B \delta + c_{34} v \delta + c_{11} a^2 + c_{22} B^2 + c_{33} v^2 + c_{44} \delta^2 \qquad \textbf{(24)}$$
 where:

c<sub>0</sub>, ..., c<sub>44</sub> are the regression coefficients;

a = 0.1 - 0.3 m is the working depth;

B = 0.8 - 1.2 m is the working width;

 $\delta$  = 13.8 – 24.2 % is the coefficient of slippage of the tractor's drive wheels;

v = 0.9 - 1.9 m/s is the working speed.

Table 1 shows the values of the independent variables and the dependent variable used in the calculation program in Mathcad, to determine the regression coefficients. After calculating the regression coefficients, the following multivariable polynomial function was obtained:

$$\begin{aligned} \text{Ft} &= 39949.216 + 20545.854\text{a} - 37064.489\text{B} - 13259.116\text{v} - 102.292\delta - 77361.486\text{aB} - 23654.742\text{av} + \\ &10539.344\text{a}\delta - 6712.583\text{Bv} + 2861.022\text{B}\delta + 1512.639\text{v}\delta - 143852.709\text{a}^2 + 10965.706\text{B}^2 - 136\text{v}^2 - \\ &205.411\delta^2 \end{aligned}$$

Next, the deviation of the traction forces calculated using relation (25) in relation to the traction forces measured in the experimental tests was calculated. Table 5 shows the values of the traction force measured during the experiments, the values of the traction force calculated using the relation (25) and the deviation.

The maximum error between the experimental data and the calculated data was 2.442%, thus resulting in a good accuracy of the proposed model. The correlation coefficient calculated in which the string X with Fi and the string Y with Fci were replaced is 0.998, demonstrating a very strong correlation between the two strings.

The coordinates of the optimal point of the interpolation function (22), isolated (which is not located on the boundary of the definition domain), were identified by solving the linear system of equations obtained by cancelling the partial derivatives of the function.

Table 5

Measured traction forces, calculated traction forces and deviation

Sample Average measured traction		Calculated average traction	Deviation [0/]	
no.	force F <sub>i</sub> [N]	force Fc <sub>i</sub> [N]	Deviation [%]	
1	17150	17109.91	0.234	
2	17560	17475.559	0.481	
3	17590	18019.533	2.442	
4	20960	21005.634	0.218	
5	21170	20997.827	0.813	
6	22100	21958.025	0.642	
7	23530	23400.92	0.549	
8	23850	23815.723	0.144	
9	24130	24261.806	0.546	
10	18240	18406.683	0.914	
11	19150	19015.664	0.701	
12	19970	19567.746	2.014	
13	21760	21938.222	0.819	
14	22060	21813.508	1.117	
15	22950	23081.479	0.573	
16	23560	23831.905	1.154	
17	24320	24358.785	0.159	
18	24480	24502.542	0.092	
19	20980	20762.76	1.035	
20	21160	21512.473	1.666	
21	21440	21371.511	0.319	
22	24280	24278.629	0.006	
23	24320	24424.82	0.431	
24	24720	24821.678	0.411	
25	26240	25983.627	0.977	
26	26840	26867.11	0.101	
27	28130	28055.923	0.263	

The second-degree polynomial function for the traction force, which interpolates the experimental data, can, however, be minimized in order to find an optimal point through a constraint minimization operation.

The function arguments are restricted in the experimental working range or at most in a slightly wider range, in the vicinity of an experimental point (sample 14: a=0.21 m, B=1.04 m, v=1.29 m/s,  $\delta=17.7$  %). The optimal coordinate point is obtained:  $a_{opt}=0.2$  m,  $B_{opt}=1$ m,  $v_{opt}=1.7333$  m/s,  $\delta_{opt}=18.33$ ,  $F_{opt}=20380.55$  N.

This optimal point is acceptable, but moving the starting point away from the experimental points may cause unacceptable results.

For the skidding of the tractor's motor wheels of 15%, the working speed v=1.4 m/s and three values of the working depth, were calculated the traction forces with relation (25), the data being presented in table 6.

Table 6
Values calculated for the traction force

Working width	Traction force [daN]			
B [m]	a = 0.1  m	a = 0.2  m	a = 0.3  m	
0.8	16238.6	20286	21456.4	
0.85	16579.1	20239.7	21023.3	
0.9	16974.4	20248.2	22655	
0.95	17424.6	20311.6	22499.4	
1	17929.6	20429.8	22398.6	
1.15	19773.6	21113.4	22425.2	
1.2	20497.9	21450.9	22543.8	

Using the data from Table 6, the graph of the variation of the traction force, depending on the working width of the plough was represented (fig. 3).

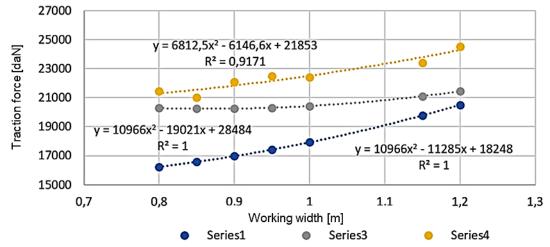


Fig. 3 - Variation of the traction force depending on the working width B, for the speed v = 1.4 m/s and for three values of the working depth: a = 0.1m (series 1), a = 0.2m (series 3) and a = 0.3 m (series 4)

Figure 3 shows that the traction force increases when increasing working depth and working width of the plough. The relations shown in figure 3 allow the calculation of the traction force for one of the working depths for any value of the working width B = 0.8 - 1.2 m.

#### **CONCLUSIONS**

The main results obtained consist in demonstrating that there are own optimal points in the working process of the tractor – plough with variable working width aggregate. In addition, the optimal points found have as abscissa not only the working speed, but also the working width, a parameter whose control defines this type of ploughs. Also, the optimality criteria used are natural, namely they have an intuitive meaning. From a physical point of view, the parametric structure of the coordinates of the optimal points leads to interesting conclusions, facilitated by the fact that these coordinates were found analytically and not numerically. Optimum points are confirmed in terms of working speed and working width in reference works in the field of agricultural machinery operation.

It is noted that of the four optimal operating points found, only one has an optimal working width as an abscissa, which minimizes the traction force (22) under conditions of a fixed productivity (21).

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