STABILITY ANALYSIS OF EQUILIBRIUM FOR TROMBE WALL SOLAR CHICK BROODER
/
NYOCHA ỊMATA MA AJA TROMB NA ỤLỌ EBE Ọ DỊ A GA-ENWE IKE INYE ỤMỤ ỌKỤKỌ ABỤRỤ ỌHỤRỤ EKPOMỌKỤ GA-ADIGIDE

Ohagwu C. J.[1)], Okonkwo W. I.[1)], Akubuo C. O.[1)], Mbah G. C.E.[2)], Njoku H. O.[3)][1]
[1)]Department of Agricultural and Bioresources Engineering, [2)]Department of Mathematics,
[3)]Department of Mechanical Engineering, University of Nigeria, Nsukka

Corresponding Author name: Chukwuemeka Jude Ohagwu
Corresponding Author Tel:+2348063507144
Email address: chukwuemeka.ohagwu@unn.edu.ng
DOI: https://doi.org/10.35633/inmateh-59-32

Keywords: *Equilibrium Analysis, Stability, Trombe wall solar brooder, optimal brooding*

ABSTRACT

Trombe wall solar chick brooder was developed for 150-200 birds' capacity. System evaluation as well as stability and equilibrium analysis were carried out. A set of linear differential equations were formulated that described the dynamic modeling of Trombe wall solar chick brooder for optimal poultry production. The descriptive models were referred to as system control. The model was transformed into Jacobian matrix and solved for stability and equilibrium. Further tests for stability using Routh-Hurwitz criterion were conducted. The solved Jacobian matrix had negative eigenvalues and as such was locally stable. Routh-Hurwitz criterion affirmed that the system model was stable. It implied, the system supported brooding of 200 birds for the 28 days of brooding operation at temperature range of 25-39°C of the brooding room.

ỤMỊ EDEMEDE
Aja tromb dị n'ụlọ si n'anyanwụ adọta ike bụ nke e wubere iji were nwee ike ịzụpụta ụmụ ọkụkọ dị 150-200 n'ọnụ ọgụgụ. Emere nnyocha sistem ya nakwa nnyocha ndigide ekpomọkụ na ndaba ụlọ ahụ iji were hụ na ọrụrụ ọrụ ka o si were kwesị. Ewepụtara usoro mgbakọ na mwepụ kọwara maka ịrụaja Tromb iji were zụọ ụmụ ọkụkọ nke ga-eme ka enwee ike na-azụpụta ọkụkọ nke ọma. Mgbakọ na mwepụ emere bụ nke ahụrụ dịkaịlekọta sistem. Emeghariri mgbakọ na mwepụ a na matris Jacob were hụ ndigide ekpomọkụ na ndaba ụlọ ahụ iji were nye ihe a chọrọ. E ji Routh-Hurwitz were mee nnyocha banyere ndigide ekpomọkụ a. Matris Jacob ewepụtara nwere mpụtara dị negetiv nke gosiri na ọdabara. Usoro Routh-Hurwitz wepụtara gosikwara na ọdabara. O na-egosi na usoro a kwadoroịzụ ụmụ ọkụkọ 200 na mkpụrụ ụbọchị 28 n'ogo okpomọkụ ụlọ dị 25-39ºC.

INTRODUCTION

Poultry industry is one the subsectors of the Agricultural sector that is contributing immensely to the economy of any nation. It requires sectorial synergy in energy supply in the value chain. Energy is required in different unit of operations (such as hatching, brooding, defeathering, etc.) in poultry production. Due to energy need in the value chain of production especially during brooding of day-old chicks, solar energy utilization technologies were most suitable. Due to obvious enormous benefits like its renewability, cheap energy source etc. These technologies as an appropriate technology were required to achieve optimal poultry brooding operation *(Okoye J.A.O, 1992).* This was essential to maximize animal health and husbandry production and it contributed to good performances. The most essential performance index/ brooding parameter to control was the chick body temperature because day old chicks were unable to regulate their body temperature post hatched *(Obioha F.C., 1992)*. The quest to proffer solution in this regard necessitated the development of Trombe wall solar chick brooder for optimal poultry production *(Echiegu E.A, 1993; 1986, Okonkwo W.I., 1993, Okonkwo W.I. and Akubuo C.O., 2001; Okonkwo W.I and*

[1] *Ohagwu C. J., Ph.D., M.Eng., B.Eng.; Okonkwo W. I., Prof. Ph.D., M.Eng., B.Eng.;Akubuo C. O., Prof., Ph.D., M.Eng., B.Eng.; Mbah G. C.E.[2)], Prof., Ph.D., M.Sc., B.Sc.; Njoku H. O., Ph.D., M.Eng., B.Eng.*

Agunwamba J.C., 1997; Okonkwo W.I., Anazodo G.U.N, Akubuo C.O., Echiegu E.A., Iloeje O.C.,1992; Okonkwo W.I. and Akubuo C.O., 2007). Experimental studies on the viability of the system have been proven *(Odo L.O., 2016)*. Perennial problems associated with poultry day-old chick brooding were solved with the aid of solar systems. Further studies have been carried out in mathematical modeling of Trombe wall solar chick brooder for optimal poultry production *(Ohagwu et.al, 2017).* The model was simulated and validated with measured data. Thus the need to examine the system further numerically for stability and equilibrium analysis was imperative. The study had shown trade-off between physically built brooder and modeled brooder (*Maria A. and Zhang C., 1997).* Therefore, modeled brooder has now become the platform for further studies.

MATERIALS AND METHODS

Passive Trombe wall solar chick brooder was developed with brooding capacity of one hundred and fifty two hundred birds (150-200) per batch as seen fig.1.0. The system adopted deep litter system of chicken brooding. The thermophysical properties, materials and dimensions of the Trombe wall solar chick brooder were shown in table 1.

PHYSICAL DESCRIPTIVE MODEL/ PARAMETRIC CONSIDERATIONS

The physical Trombe wall *(Trombe et.al., 1977)* solar chick brooder was dimensioned (3m x 2.2m x 3m) with brooding area as 6.6m^2 *(Okonkwo W.I., Akubuo C.O., 2007)* The building materials used for the construction were materials accessed locally in the tropics, namely: solid cement blocks, acrylic transparent glazing, ceiling board, zinc, pebble stone (limestone chippings), black paint and wooden door. The physical component model of the Trombe wall solar chick brooder studied is shown in Figure 1 and was segmented into components namely: (a) solar energy collector (vented)-the Trombe wall (b) pebble bed collector (vented)- pebble bed bin (c) thermal heat storage wall (vented)-Trombe wall and pebble bed bin (d) Air Chimney (e) poultry brooding room (f) sensible heat from the birds stocked and exit of heat and water vapour transmission in the poultry room through the chimney. But the basic component parts were shown below.

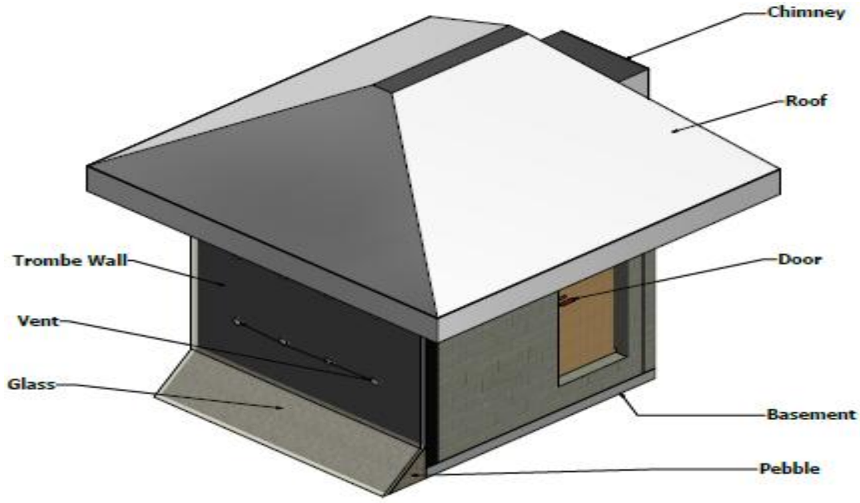

Fig. 1 - Three Dimensional View of Trombe Wall Solar Chick Brooder with the Component Parts

The effect of solar radiation incidence and utilization on the physical model - solar collector component is illustrated in figure 2. The structure is oriented facing due south. During the sunrise of the day, radiant energy from the sun is absorbed, stored and concentrated on the collector surface through the glaze. This energy is converted to heat energy which was absorbed on the surface. The absorbed energy caused an increase in the internal energy of the Trombe wall thereby increasing the temperature of the collector. The energy got stored and by conduction (assumed-one directional) transported to inner surface of the Trombe wall. Factors that determined solar radiation intensity received by the Trombe wall included *(Duffie J.A. and Beckmann W.A., 1991)*: Earth-sun distance (G_{sn}) which depended on the number of the day(n) in a year (Jan1=1, Dec.31=365), Air mass (m), insolation (H), total/global solar radiation (beam and diffuse radiation), sometimes, reflected radiation and is measured against horizontal surface, solar altitude angle (m),solar time and solar angle(θ) which is dependent on latitude ϕ, declination δ, slope, β solar hour angle ω , angle of incidence θ, zenith angle θ_z, solar altitude angle α_s, surface azimuth angle γ, solar azimuth angle γ_s. These factors were used in determining the quantity of heat energy from the sun used for brooding operation.

Table 1

Thermo-physical Properties, Materials and Dimensional parts of Trombe wall Solar Brooder

Trombe wall glaze (perplex)	Units
Emissivity	1180
Absorptivity	0.05
Specific heat capacity (J/kg/K)	1470
Transmissivity	0.75
Thickness(m)	0.003
Height (m)	3
Width	3
Trombe wall (Masonry wall)	
Emissivity	0.86
Absorptivity	0.95
Thickness (m)	0.40
Radius of vents	0.015
Width and width of the air gap	3
Sunspace width (D_{gap})	0.003
Thermal diffusivity	$K_{Tw}/(\rho_{Tw}Cp_{Tw}T)$
Height (m) and height of the air gap (m)	3
Thermal Conductivity (K) (W/m^2)	0.1
Specific heat capacity (J/Kg/ ^{0}C)	920
Density (Kg/m^3)	2240
Pebble bed bin	
Emissivity	0.88
Absorptivity	0.95
Height between inlet and outlet	0.5
Brooder (Deep litter)	
Overall heat loss (J/Kg/Kg)	30
Width (m)	2.2
Length of door	1.83
Width of door	0.3
Thermal conductivity of door	0.16
Door thickness (m)	0.15
Area	A_{Tw} x (4)
Area of the chimney outlet vent	0.0009
Discharge	0.62
Air	
Specific heat capacity (K/Kg/Kg)	1005
Viscosity (Kg/m^3)	1.857
Density (Kg/m^3)	1.177
Prandtl Number	0.713
Thermal conductivity (W/m/K)	0.02544
Thermal diffusivity (m^2/s)	2.213 x10^{-5}

MATHEMATICAL FLOW MODEL OF TROMBE WALL SOLAR ENERGY BROODER SYSTEM

This numerically described the energy trend flow across the system component of the brooder. The energy transport and transformation across the component parts are as follows: global solar radiation, glaze of Trombe wall and pebble bed bin, Trombe wall, pebble bed bin, brooding room, chicks and other sources of heat losses such as roof, walls, chimney and door. The global solar radiation component regarded as diffused radiation is part of the heat losses such as reflected rays(r) and ground reflected rays (gr) are part of radiant energy considered in the model. Neglecting the initial temperature of the brooding room before the birds were stocked with respect to heat gain from the Trombe wall and pebble bed bin, the system process models were explained thus: solar radiation intensity (sun) incident on the vertical glaze (G_1) of the Trombe wall from sunrise to sunset, transmitted heat energy to the Trombe wall (T_w). Also solar radiation was incident on the inclined glaze (G_2) of the pebble bed bin (Pb) from sunrise to sunset, transmitted heat energy to the pebble bed bin.

Of course, solar radiations are characterized by losses due to reflection(r) and ground reflected rays (gr) as well as solar flux incident on the collector (beam and diffused radiation). These heat energy/flux were stored on Trombe wall (T_w) and were convectionally released the heat to the brooding room where day old chicks were stocked through the upper and lower vents and that of the lower vents of the pebble bed bin and conductively across the wall. The heat energy stored on Trombe wall (T_w) and pebble bed bin exchanged heat to each other via conduction and convection by thermo-circulation. There were thermal interactions between Trombe wall (T_w), pebble bed bin (Pb) and N-chicks, other heat losses were through chimney, other walls, roof etc. Fig. 2 showed the flow diagram of the mathematical model equations formulated.

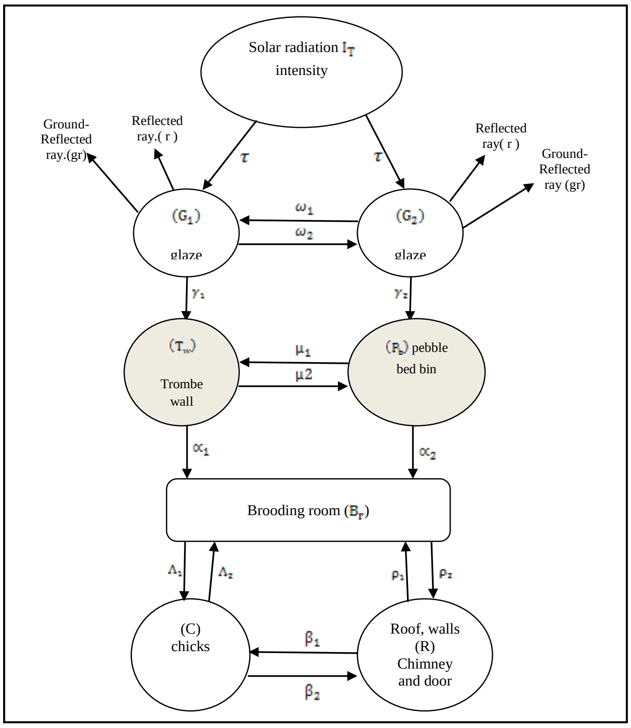

Fig. 2 - Mathematical Flow Model Diagram of Trombe wall Solar Energy Chick Brooder

As a result of thermo-circulation and thermal losses in and across the brooding room, and in order to maintain optimal energy balance in the brooding room, temperature of the brooding room must be greater than the ambient temperature or heat gained through the roof (R). If the ambient temperature (Ta) of the surroundings were greater than the brooding room temperature (T_{Br}) then heat losses occurred through the other walls, roof and other orifices. If the temperature of the brooding room was greater than the temperature of the walls, in other words, other walls absorbed the temperature gradient. Temperature fluctuation/swing between the environment and the brooding room was such that the overall objectives were to provide optimal heat (temperature) required for brooding of chicks within the environmental atmospheric pressure. The quantity of heat gained /lost $Q_{G1,}Q_{G2}, Q_{Tw}, Q_{Pb}, Q_{Br}, and\ Q_R$ across the system to the brooding room and exit through the chimney created potential brooding energy for the birds. From the mathematical flow model diagram in fig. 2, at any given time, heat gains were expressed additively while heat losses were by subtraction. Therefore, to determine the quantity of heat gain across the Trombe wall glaze at any given time (t), the model equation was given as:

$$\frac{dQ_{G1}}{dt} = \tau I_T + \omega_1 dQ_{G2} - \gamma_1 dQ_{G1} - rdQ_{G1} - grdQ_{G1} - \omega_2 dQ_{G1} \quad (1)$$

where dQ_{G1} *is the differential heat loss from the glaze.*

Also, to determine the quantity of heat gain across the pebble bed bin glaze at any given time (time), the model equation was given as:

$$\frac{dQ_{G2}}{dt} = \tau dI_T + \omega_2 dQ_{G1} - \omega_1 dQ_{G2} - \gamma_2 dQ_{G2} - rdQ_{G2} + grdQ_{G2} \quad (2)$$

For the quantity of heat gain at any time (t) on the Trombe wall, we have the model equation as:

$$\frac{dQ_{TW}}{dt} = \gamma_1 dQ_{G1} + \mu_1 dQ_{Pb} - \mu_2 dQ_{TW} - \alpha_1 dQ_{TW} \quad (3)$$

For the quantity of heat gain at any time (t) on the pebble bed bin storage, we have the model equation as:

$$\frac{dQ_{Pb}}{dt} = \gamma_2 dQ_{G2} + \mu_2 dQ_{TW} - \mu_1 dQ_{Pb} - \alpha_2 dQ_{Pb} \quad (4)$$

To determine the temperature of the chick at any time (t) in the brooding room, we have the model equation as:

$$\frac{dT_C}{dt} = \lambda_1 dT_{Br} + \beta_1 dT_R - \lambda_2 dT_C - \beta_2 dT_C \quad (5)$$

Temperature of the chick can only be maintained, if $dQ_{Br} > dT_A$ where dT_A= ambient temperature, then $\lambda_2 = \beta_1 = 0$, dT_C = change in temperature of the chicks and dQ_R = change in quantity of the heat or temperature of the roof, n-walls and door.

To determine the quantity of heat loss from the roof, walls, chimney and door at any time (t) of the brooding room, we have the model equation as:

$$\frac{dQ_R}{dt} = \rho_2 dQ_{Br} + \beta_2 dT_C - \rho_1 dQ_R - \beta_1 dQ_R \quad (6)$$

Meanwhile, to determine the quantity of heat in the brooding room at any time (t) to keep the chicks warm, we have the model equation as:

$$\frac{dQ_{Br}}{dt} = \alpha_1 dQ_{TW} + \alpha_2 dQ_{pb} + \lambda_2 dT_C + \rho_1 dQ_R - \lambda_1 dQ_{Br} - \rho_2 dQ_{Br} \quad (7)$$

Equation (7) holds, if $\lambda_1 = 0\, if dQ_{Br} > dT_C and \rho_2 = 0, if dQ_{Br} > dT_A$

- **EQUILIBRIUM STATE ANALYSIS OF THE SYSTEM MODEL**

For the system model to achieve stability among the heat contributing component factors affecting thermal transport/the operation of the brooding process (maintaining optimal temperature for chick brooding), the factors like the change in the temperatures of the Trombe wall glazing, pebble bed bin glazing, Trombe wall, pebble bed bin, brooding room, and chicks with respect to time, neglecting heat losses via roof, walls, chimney and door supported the brooding operation. Therefore, the Equilibrium State Analysis of the System Model was given in equation (8)

$$\frac{dT_{TW}}{dt} = \frac{dT_C}{dt} = \frac{dT_{Br}}{dt} = \frac{dT_{pb}}{dt} = \frac{dT_{G1}}{dt} = \frac{dT_{G2}}{dt} = \frac{dT_R}{dt} = 0 \quad (8)$$

Ultimately, for the system to be in equilibrium, the system component parts that provide the enthalpy must be equal such that: change in temperature of the Trombe wall is equal to change in temperature of the chicks and is equal to change in temperature of the brooding room and is equal to change in temperature of pebble bed bin and is equal to change in temperatures of glazing of the Trombe wall and pebble bed bin as well as change in the temperatures of the roof, chimney, walls and door. Relying on equations (1) to (7) to derive the equilibrium stability of each system component parts that contributes to these processes, we have the following expressions:

For Trombe wall temperature, referring to equation (3), it is expressed as:

$$\frac{\partial T_{TW}}{\partial t} \xrightarrow{yields} 0, \Rightarrow \gamma_1 dT_{G1} + \mu_1 dT_{Pb} - \mu_2 dT_{TW} - \alpha_1 dT_{Tw} = 0$$

$$\gamma_1 dT_{G1} + \mu_1 dT_{Pb} - (\mu_2 + \alpha_1) dT_{Tw} = 0 \Rightarrow dT_{TW} = \frac{\gamma_1 dT_{G1} + \mu_1 dT_{Pb}}{\alpha_1 + \mu_2} \quad (9)$$

Heat gained by the system, even if heat supply from Trombe wall yielded as low as zero, equation (9) still supported the system to brood the chicks.

For temperature of *N* chicks, refer to equation (5), it is expressed as:

$$\frac{dT_C}{dt} \xrightarrow{yields} 0, \quad \Rightarrow \lambda_1 dT_{Br} + \beta_1 dT_R - \lambda_2 dT_C - \beta_2 dT_C = 0$$ see the fig.2.

$$\lambda_1 dT_{Br} + \beta_1 dT_R - (\lambda_2 + \beta_2) dT_C = 0 \Rightarrow (\lambda_2 + \beta_2) dT_C = \lambda_1 dT_{Br} + \beta_1 dT_R$$

$$\Rightarrow dT_C = \frac{\lambda_1 dT_{Br} + \beta_1 dT_R}{(\lambda_2 + \beta_2)} \quad (10)$$

Heat gained by the system, even if heat supply from chicks yielded as low as zero, equation (10) still supported the system to brood the chicks.

For temperature of brooding room, referring to equation (7), it is expressed as:

$$\frac{\partial T_{Br}}{\partial t} \xrightarrow{yields} 0, \Rightarrow \alpha_1 dT_{TW} + \alpha_2 dT_{pb} + \lambda_2 dT_C + \rho_1 dT_R - \lambda_1 dT_{Br} - \rho_2 dT_{Br} = 0$$

$$\alpha_1 dT_{TW} + \alpha_2 dT_{pb} + \lambda_2 dT_C + \rho_1 dT_R - (\lambda_1 + \rho_2) dT_{Br} = 0 \Rightarrow (\lambda_1 + \rho_2) dT_{Br} = \alpha_1 dT_{TW} + \alpha_2 dT_{pb} + \lambda_2 dT_C + \rho_1 dT_R$$

$$dT_{Br} = \frac{\alpha_1 dT_{TW} + \alpha_2 dT_{pb} + \lambda_2 dT_C + \rho_1 dT_R}{\lambda_1 + \rho_2} \quad (11)$$

Heat gained by the system, even if heat supply from brooder yielded as low as zero, equation (11) still supported the system to brood the chicks.

For temperature of pebble bed bin, referring to equation (4), it is expressed as:

$$\frac{dT_{Pb}}{dt} = \xrightarrow{yields} 0, \Rightarrow \gamma_2 dT_{G2} + \mu_2 dT_{TW} - \mu_1 dT_{Pb} - \alpha_2 dT_{Pb} = 0$$

$$\gamma_2 dT_{G2} + \mu_2 dT_{TW} - (\mu_1 + \alpha_2) dT_{Pb} = 0 \Rightarrow (\mu_1 + \alpha_2) dT_{Pb} = \gamma_2 dT_{G2} + \mu_2 dT_{TW} \Rightarrow dT_{pb} = \frac{\gamma_2 dT_{G2} + \mu_2 dT_{TW}}{(\mu_1 + \alpha_2)} \quad (12)$$

Heat gained by the system, even if heat supply from pebble bed yielded as low as zero, equation (12) still supported the system to brood the chicks.

For temperature of Trombe wall glazing, referring to equation (1), it is expressed as:

$$\frac{dT_{G1}}{dt} = \xrightarrow{yields} 0, \Rightarrow \tau dI_T + \omega_1 dT_{G2} - \gamma_1 dT_{G2} - \gamma_1 dT_{G1} - r dT_{G1} - gr dT_{G1} - \omega_2 dT_{G1} = 0$$

$$\tau dI_T + (\omega_1 - \gamma_1) dT_{G2} - (\gamma_1 + r + gr + \omega_2) dT_{G1} = 0$$

$$\frac{\tau dI_T + (\omega_1 - \gamma_1) dT_{G2}}{(\gamma_1 + r + gr + \omega_2)} = dT_{G1} \quad (13)$$

For temperature of pebble bed bin glazing, referring to equation (2), it is expressed as:

$$\frac{dT_{G2}}{dt} = \xrightarrow{yields} 0, \Rightarrow \tau dI_T + \omega_2 dT_{G1} - \omega_1 dT_{G2} - \gamma_2 dT_{G2} - r dT_{G2} + gr dT_{G2} = 0$$

$$\tau dI_T + \omega_2 dT_{G1} - (\omega_1 + \gamma_2 + -gr) dT_{G2} = 0 \Rightarrow dT_{G2} = \frac{\tau dI_T + \omega_2 dT_{G1}}{(\omega_1 + \gamma_2 + r - gr)} \quad (14)$$

For temperature of roof, walls, chimney and door refer to equation (6), it is expressed as:

$$\frac{dT_R}{dt} = 0, \Rightarrow \rho_2 dT_{Br} + \beta_2 dT_C - \rho_1 dT_R - \beta_1 dT_R = 0$$

$$\rho_2 dT_{Br} + \beta_2 dT_C - (\rho_1 + \beta_1) dT_R = 0$$

$$dT_R = \frac{\rho_2 dT_{Br} + \beta_2 dT_C}{(\rho_1 + \beta_1)} \quad (15)$$

STABILITY/STEADY STATE ANALYSIS OF THE SYSTEM MODEL

From the derived model equations (1) to (7), the stability/steady state of the components parts that contribute heat/ temperature change to the brooding system can be mathematically presented in a matrix form given as:

$$\begin{pmatrix} T_{G1} & T_{G2} & T_{TW} & T_{pb} & T_{Br} & T_C & T_R \\ -(\omega_2 + \gamma_1 + r + gr) & \omega_1 & 0 & 0 & 0 & 0 & 0 \\ \omega_2 & -(\omega_1 + \gamma_2 + r + gr) & 0 & 0 & 0 & 0 & 0 \\ \gamma_1 & 0 & -(\alpha_1 + \mu_1) & \mu_2 & 0 & 0 & 0 \\ 0 & \gamma_2 & +\mu_2 & -(\alpha_1 + \mu_2) & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & +\alpha_2 & -(\lambda_1 + \rho_2) & \lambda_2 & -\rho_1 \\ 0 & 0 & 0 & 0 & +\lambda_1 & -(\lambda_2 + \beta_2) & +\beta_1 \\ 0 & 0 & 0 & 0 & +\rho_2 & +\beta_2 & -(\rho_1 + \beta_1) \end{pmatrix}$$

The matrix above is transformed to Jacobian matrix; solving this equation might be difficult, but test for the stability condition is stated as follows: we only need the sign of the eigenvalues. The steady state point is stable if Re (λ) < 0 for all λ: The stability condition is established by the following Routh-Hurwitz criteria.

Given the characteristic equation

$$\lambda^k + a_1 \lambda^{k-1} + a_2 \lambda^{k-2} + \cdots + a_k = 0$$

From the following k Hurwitz matrices:

$$H_1 = (a_1) \quad ; H_2 = \begin{pmatrix} a_1 & 1 \\ a_3 & a_2 \end{pmatrix} \quad ;$$

$$H_3 = \begin{pmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{pmatrix} \quad ; \dots \; ; \; H_3 = \begin{pmatrix} a_1 & 1 & 0 & \dots & 0 \\ a_3 & a_2 & a_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a_k \end{pmatrix}$$

The steady state is stable that is $Re(\lambda) < 0 \; for \; all \; \lambda$ if $det \; H_j \; \geq 0 \; for \; all \; j = 1,2, \dots, k$ If the eigenvalues of J all have real parts less than zero, then the steady state is stable.

- If at least one of the eigenvalues of J has real part greater than zero, then the steady state is unstable.
- If at least one of the eigenvalues of J has real part equal to zero then no conclusion can be made from the linear analysis. We have a borderline case between stability and instability. In these cases, nonlinear terms need to be considered *(Nazim M. and Hazrat M.D, 2018)*.

RESULTS AND DISCUSSIONS

The results for stability were presented as:

$$J = \begin{pmatrix} -(\omega_2 + y_1 + r + gr) & \omega_1 & 0 & 0 & 0 & 0 & 0 \\ \omega_2 & -\omega_2 + y_2 + r + gr & 0 & 0 & 0 & 0 & 0 \\ \gamma_1 & 0 & -(\mu_2 + \alpha_1) & \mu_1 & 0 & 0 & 0 \\ 0 & \gamma_2 & \mu_2 & -(\mu_1 + \alpha_2) & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & \alpha_2 & \lambda_2 & \lambda_2 & \rho_1 \\ 0 & 0 & 0 & 0 & -(\lambda_2+\beta_2) & -(\lambda_2+\beta_2) & \beta_1 \\ 0 & 0 & 0 & 0 & \beta_2 & \beta_2 & -(\rho_1 + \beta_1) \end{pmatrix} = 0$$

Neglecting 7th column of the matrix since it is heat gain from the roof T_R , which is minor; the matrix is reduced to 7x6 matrices given as:

$$J = \begin{pmatrix} -(\omega_2 + y_1 + r + gr) & \omega_1 & 0 & 0 & 0 & 0 \\ \omega_2 & -\omega_2 + y_2 + r + gr & 0 & 0 & 0 & 0 \\ \gamma_1 & 0 & -(\mu_2 + \alpha_1) & \mu_1 & 0 & 0 \\ 0 & \gamma_2 & \mu_2 & -(\mu_1 + \alpha_2) & 0 & 0 \\ 0 & 0 & \alpha_1 & \alpha_2 & \lambda_2 & \lambda_2 \\ 0 & 0 & 0 & 0 & -(\lambda_2+\beta_2) & -(\lambda_2+\beta_2) \\ 0 & 0 & 0 & 0 & \beta_2 & \beta_2 \end{pmatrix} = 0$$

$A_1 = \omega_2 + \gamma_1 + r + gr, \; A_2 = \omega_1 + \gamma_2 + r + gr,$

$A_3 = \mu_2 + \alpha_2, A_4 = \mu_1 + \alpha_2, A_5 = \lambda_1 + \rho_2 \,,$

$$A_6 = \lambda_1 \lambda_2 + \lambda_2 \beta_2 (\lambda^* + A_1)(\lambda^* + A_2)(\lambda^* + A_3)(\lambda^* + A_4)(\lambda^* + A_5)(\lambda^* + A_6) = 0$$

$\Rightarrow \lambda^{*6} + \beta_1 \lambda^{*5} + \beta_2 \lambda^{*4} + \beta_3 \lambda^{*3} + \beta_4 \lambda^{*2} + \beta_5 \lambda^* + \beta_6 = 0$ but, $B_1 = A_1 + A_2 + A_3 + A_4 + A_5 + A_6$

$$B_2 = A_6(A_1 + A_2 + A_3 + A_5) + A_5(A_1 + A_2 + A_3 + A_4) + A_4(A_1 + A_2 + A_3) + A_3(A_1 + A_2) + A_1 + A_2$$

$$B_3 = A_6(A_1A_2 + A_1A_3 + A_1A_4 + A_1A_5 + A_2A_3 + A_2A_4 + A_2A_5 + A_3A_4 + A_3A_5 + A_4A_5) + A_5(A_1A_2 + A_1A_3 + A_1A_4 + A_2A_3 + A_2A_4 + A_3A_4) + A_1A_2 + A_1A_2A_3 + A_2A_3A_4$$

$$B_4 = A_6(A_1A_2A_3 + A_1A_2 + A_1A_2A_5 + A_1A_3A_4A_5 + A_1A_3A_5 + A_1A_4A_5 + A_2A_3A_4 + A_2A_3A_5 + A_2A_4A_5 + A_3A_4A_5) + A_5(A_1A_2 + A_1A_2A_3 + A_1A_3A_4 + A_2A_3A_4) + A_1A_2A_3A_4$$

$$B_5 = A_6(A_2A_3A_4A_5 + A_1A_3A_4A_5 + A_1A_3A_4A_5 + A_1A_2A_5 + A_1A_2A_3A_5 + A_1A_2A_3A_4) + A_5(A_1A_2A_3A_4) B_6 = A_1A_2A_3A_4A_5A_6$$

Now we find the eigenvalues by forming the characteristics equation given by $|\mathrm{A} - \lambda I| = 0,$ where *A* is replaced with *J*.

Therefore, the eigenvalues are:

$$\lambda_1^* = -(\omega_2 + \gamma_1 + r + gr), \lambda_2^* = -(\omega_1 + \gamma_2 + r + gr), \lambda_3^* = -(\mu_2 + \alpha_1), \lambda_4^* = -(\mu_1 + \alpha_2),$$
$$\lambda_5^* = -(\lambda_1 + \rho_2), \qquad \lambda_6^* = -(\lambda_2 + \beta_2), \lambda_7^* = -(\rho_1 + \beta_1)$$

Since, all the eigenvalues are negative; we can conclude that the stability of equilibrium state of the model is locally stable. Further prove of stability using Routh-Hurwitz criteria, given the polynomial $H(\lambda) =$

$\lambda^n + \beta_1\lambda^{n-1} + \beta_{n-1} + \beta_n$. where the coefficients β_i are real constants $i = 1, \dots, n,$ define the n Hurwitz (T) matrices using coefficients β_i of the characteristic polynomial.

$$\begin{vmatrix} -(\omega_2+y_1+r+gr) & \omega_1 & 0 & 0 & 0 & 0 & 0 \\ \omega_2 & -\omega_2+y_2+r+gr & 0 & 0 & 0 & 0 & 0 \\ \gamma_1 & 0 & -(\mu_2+\alpha_1) & \mu_1 & 0 & 0 & 0 \\ 0 & \gamma_2 & \mu_2 & -(\mu_1+\alpha_2) & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & \alpha_2 & \lambda_2 & \lambda_2 & \rho_1 \\ 0 & 0 & 0 & 0 & -(\lambda_2+\beta_2) & -(\lambda_2+\beta_2) & \beta_1 \\ 0 & 0 & 0 & 0 & \beta_2 & \beta_2 & -(\rho_1+\beta_1) \end{vmatrix} = 0$$

$$(\omega_2+\gamma_1+r+gr+\lambda^*)(\omega_1+\gamma_2+r+gr+\lambda^*)(\mu_2+\alpha_1+\lambda^*)(\mu_1+\alpha_2+\lambda^*)(\lambda_1+\rho_2+\lambda^*)(\lambda_2+\beta_2+\lambda^*)(\lambda_1\lambda_2) = 0$$

$$T_n = \begin{vmatrix} B_1 & 1 & 0 & 0 & 0 & \cdots & 0 \\ B_3 & B_2 & B_1 & 1 & 1 & \cdots & 0 \\ B_5 & B_4 & B_3 & B_2 & B_1 & \cdots & 0 \\ B_7 & B_6 & B_5 & B_4 & B_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & B_n \end{vmatrix}$$

Where $B_i = 0$ if j > n. all the roots of the polynomial H (λ) are negative or have negative real parts if and only if the determinants of all Hurwitz matrices are positive. $det\ (T_j) > 0, j = 1, 2, \cdots, n$.

For the characteristics polynomial in (*), when n = 6, the Routh – Hurwitz criteria (*Negrean I., 2015*) are: $B_1 > 0,\ B_2 > 0,\ B_3 > 0,\ B_4 > 0,\ B_5 > 0, B_6 > 0 det\ (T_1) = B_1 > 0. det\ (T_2) = \begin{pmatrix} B_1 & 1 \\ B_3 & B_2 \end{pmatrix} = B_1B_2 > 0 det\ (T_3) = \begin{pmatrix} B_1 & 1 & 0 \\ B_3 & B_2 & B_1 \\ B_5 & B_4 & B_3 \end{pmatrix} = B_1B_2B_3 - {B_3}^2 > 0 \Rightarrow\ B_1B_2 - B_3 > 0$

$$B_1\left(B_1B_3B_4 - {B_2}^2B_5 - B_1{B_4}^5 - B_1B_2B_6 + 2B_4B_5 - B_3B_6\right) \det(T_4) = \begin{pmatrix} B_1 & 1 & 0 & 0 \\ B_3 & B_2 & B_1 & 1 \\ B_5 & B_4 & B_3 & B_2 \\ 0 & B_6 & B_5 & B_4 \end{pmatrix} = - B_3\,(B_3B_4 - B_2B_5) - {B_2}^2 > 0$$

$$\det(T_5) = \begin{pmatrix} B_1 & 1 & 0 & 0 & 0 \\ B_3 & B_2 & B_1 & 1 & 0 \\ B_5 & B_4 & B_3 & B_2 & B_1 \\ 0 & B_6 & B_5 & B_4 & B_3 \\ 0 & 0 & 0 & 6 & B_5 \end{pmatrix}$$

$$= B_1\left(B_2B_3B_4B_5 - B_2{B_3}^2B_6 - {B_2}^2{B_5}^2 + B_1B_2B_5B_6 + B_1{B_4}^2B_5 + B_1B_3B_4B_6 + B_1B_2B_5B_6 - {B_1}^2{B_6}^2 + B_4{B_5}^2 - 2B_3B_5B_6 + B_4{B_5}^2\right) - \left({B_3}^2B_4B_5 + {B_3}^3B_6 + B_2B_3{B_5}^2 - B_3{B_5}^2B_6\right) > 0$$

$$det\ (T_5) = \begin{pmatrix} B_1 & 1 & 0 & 0 & 0 & 0 \\ B_3 & \beta_2 & B_1 & 1 & 0 & 0 \\ B_5 & B_4 & B_3 & B_2 & B_1 & 1 \\ 0 & B_6 & B_5 & B_4 & B_5 & B_2 \\ 0 & 0 & 0 & B_6 & B_5 & B_4 \\ 0 & 0 & 0 & 0 & 0 & B_6 \end{pmatrix}$$

$$= B_1\left(B_2B_3B_4B_5 - B_2{B_3}^2{B_6}^2 - {B_2}^2{B_5}^2 + B_1B_2B_5B_6 + B_1B_2B_5{B_6}^2 - B_1{B_4}^2B_5B_6 + B_1B_3{B_4}^2{B_6}^2 + B_1B_2B_5{B_6}^2 - B_1{B_6}^3 + 2B_4{B_5}^2B_6 - 2B_3B_5{B_6}^2\right) - \left({B_3}^2B_4B_5B_6 - {B_3}^3{B_6}^2 - B_2B_3{B_5}^2B_6 + {B_5}^3B_6\right) > 0$$

From the above solutions, all the determinants of the Routh-Hurwitz matrices are positive, this implies that all the eigenvalues of the Jacobian matrices have negative real part to which further validate earlier result. The model equations above were simulated and validated using measured data. Fig.3: showed graph of measured and predicted data of the temperatures of the Trombe wall inner surface and the brooding room temperature of the Trombe wall solar chick brooder. The graph (fig.3) showed that the temperature varies with definite pattern (sinusoidal) for both measured and predicted for the days simulated with minimum average temperature difference of less than 3°C. The graph validates the mathematical matrix model of the system.

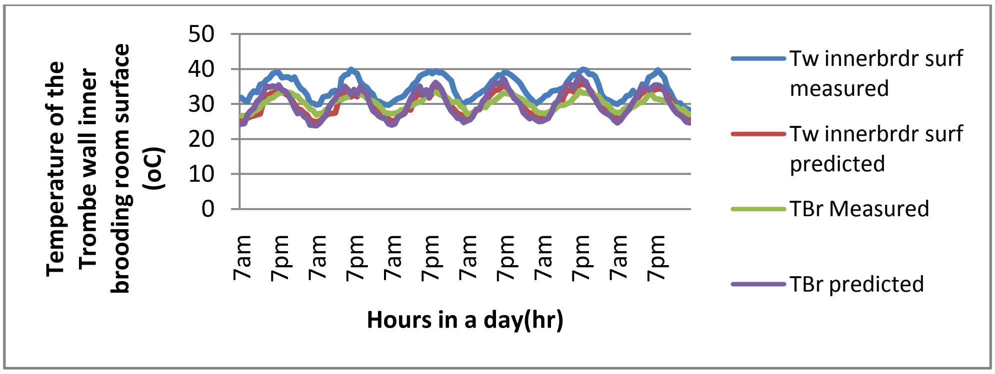

Fig. 3 - The relationship between Measured and Predicted Temperature of the brooding room and Trombe wall inner brooding surface Temperature

Therefore, the stability of equilibrium of the model is locally stable. Optimal chick stocking capacity is a function of chicks' metabolism and growth rate/age, ultimately to avoid overcrowding which may lead to cannibalism, heat stroke, disease outbreak etc. (*Harris G.C.1976*). For these reasons, the effects of stocking capacity (200birds) were studied for 28 days. Fig. 4: showed the graphical relationship between the temperature of the brooding room and Trombe wall room surface for 200 birds. The graph was broadened in Figure 5 and 6 to show the relationship for first two days and last two days of simulation (28 days). For the first two days of brooding, the system temperature in the morning (around 6-7 am) was 24.5 - 25^{o}C (room temperature), and also there was an interaction (heat energy release) between the temperatures of the Trombe wall room surface and the temperature of the brooding room with 200 birds stocked whereby the Trombe wall room surface releases heat energy required for brooding in the brooding room from 5pm to 10-11am (off- pick period). This was the pattern of energy release throughout the brooding period. The system provided variation of minimum temperature swing average of 25.5^{o}C and maximum of 39^{o}C based on solar radiation and environmental temperature of brooding. However, the Trombe wall and pebble bed bin were storing and dissipating heat energy in the day time of solar radiation.

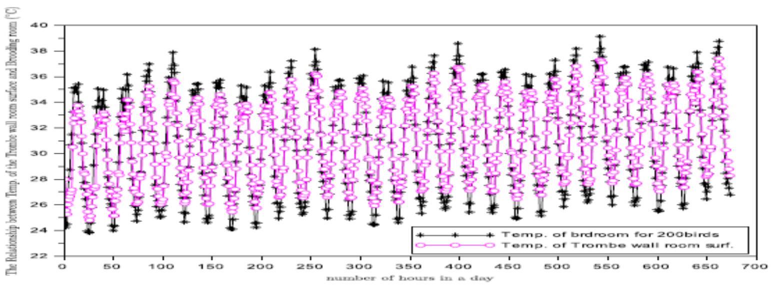

Fig. 4 - The relationship between Temperature of the Brooding room and Trombe wall room surface for 200 birds in 28 days

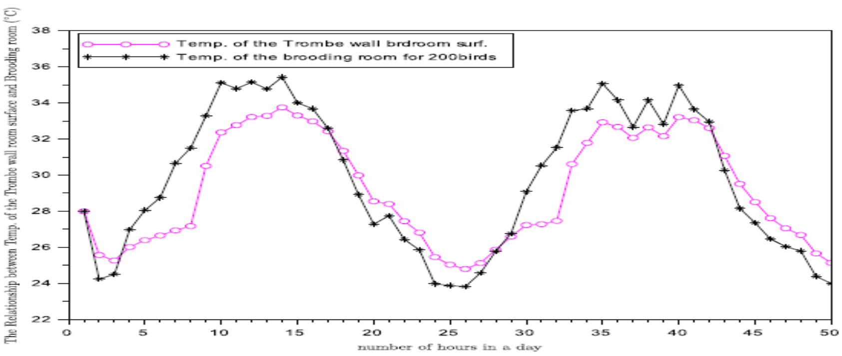

Fig. 5 - The relationship between Temperature of the Brooding room and Trombe wall room surface for 48 hours in day 2 (two) of 28 days of brooding

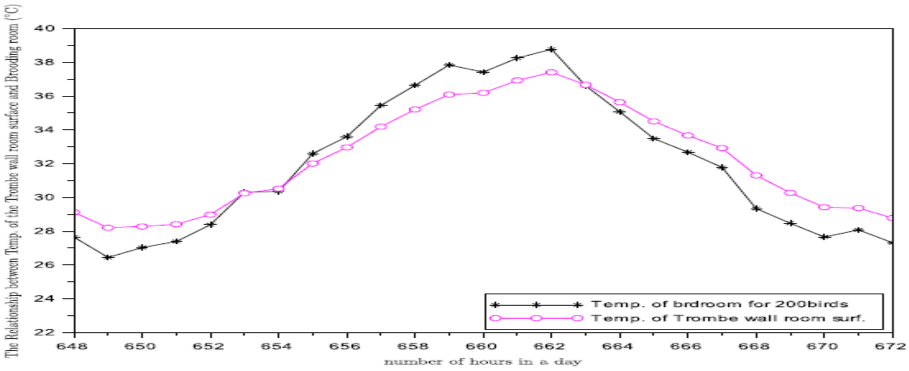

Fig. 6 - The relationship between Temperature of the Brooding room and Trombe wall room surface for 24 hours in day 28 (twenty-eight) of 28 days of brooding

These validate the component models that showed the brooder ability of the system model.

CONCLUSIONS

This work has established solar energy utilization in poultry chick brooding operation (production) with the development stability and equilibrium analysis of the Trombe wall solar chick brooder. The mathematical models formulated that describe the system model were in equilibrium with the system component model that makes up the developed model of the Trombe wall solar chick brooder. The models were locally stable at the instance of solving the Jacobian matrix of the Trombe wall solar chick brooder. It was proven further using the Routh-Hurwitz criteria for steady state stability which showed that the model was stable. The model supported the brooding of 200 birds given the dimension of the brooder.

REFERENCES

[1] Duffie J.A., Beckman W. A, (1991), *Solar Engineering of Thermal Processes*, Wiley;

[2] Echiegu E.A, (1986), The Development and Testing of a Passive Solar Energy Heated Poultry Brooder, *unpublished M. Eng., thesis, Dept. of Agricultural Engineering,* University of Nigeria, Nsukka;

[3] Echiegu E.A., (1993), Application of Solar Heat to Chicken Brooding", *Workshop paper, solar drying technologies for Agric. and Industries*, Energy Research Centre, University of Nigeria, Nsukka;

[4] Harris G.C., (1976), Limited Space Brooding in Optimum Brooding Conditions physio. *Bioenergetics Study Group* vol.12, pp.102-109;

[5] Maria A., Zhang L., (1997), Probability Distributions, Version 1.0 July 1997 Monograph, *Department of Systems Science and Industrial Engineering, SUNP at Binghamto*n, Binghamton, NY13902;

[6] Nazim M. Hazrat, A. Md (2018), A New Algorithm to Control Dynamic Stability of Higher-Order Systems, *Control System Computing and Engineering (ICCSCE 2018) 8th IEEE International Conference on*, pp. 53-58;

[7] Negrean I., (2015), Energies of Acceleration in Advanced Robotics Dynamics. *Applied Mechanics and Materials,* vol. 762, pp.67-73, Trans Tech publications;

[8] Obioha F.C. (1992). A Guide to Poultry Production in the Tropics, ACENA Publishers Enugu, Nigeria;

[9] Odo L.O (2016), Design, Construction and Performance Evaluation of a Passive solar Energy Heated Poultry Chick Brooder for Education, power and Employment for changing communities, *Journal of qualitative Education* Vol. 12, pp. 91-98;

[10] Ohagwu C.J., (2017), Mathematical Modeling of Trombe Wall Solar Chick Brooder for Optimal Poultry Production, *ABE, UNN, PhD. thesis* Research Work;

[11] Okonkwo W.I., (1993), Performance Evaluation of a Medium Scale Passive Solar Energy Brooding System, *Nigerian Journal of Solar Energy,* vol.12, pp.51-60;

[12] Okonkwo W.I. and Akubuo, C.O., (2001), Thermal Analysis and Evaluation of Heat Requirement of a Passive Solar Energy Poultry Chick Brooder. *Proceedings of the NIAE*.Vol.23 pp. 374-385.

[13] Okonkwo W.I. and Akubuo, C.O., (2007), Trombe Wall System for Poultry Brooding. *International Journal of Poultry Science,* vol. 6(2), pp. 125-130;

[14] Okonkwo W.I., AgunwambA J.C., (1997), Socio-Economic Impact of Solar Brooding System, *Proceedings International Conf. on Power Systems Operation and Planning* Abidjan;

[15] Okonkwo W.I., Anazodo G.U.N, Akubuo C.O., Echiegu E.A.,Iloeje, O.C (1992). The UNN Passive Solar Poultry Chick Brooder - Further Improvement and Preliminary Test" *Nig. J. Solar Energy*, vol.11, pp.32-40;

[16] Okoye J.A.O, (1992), Poultry Production in Nigeria, Prospects and Challenges. In: *Book of Proceedings. Workshop on improved disease diagnosis, health, nutrition and risk management practices in Poultry Production Efficiency (WIDRP)*, Ahmadu Bello University, Zaria, Nigeria, pp. 22-34;

[17] Trombe F., Robert J.F., Cabanat M., Sesolis B., (1977), Concrete Walls to Collect and Hold Heat. *Solar energ*y 2 (8) 13-19.